Infinite-Maturity Public Debt and
the Fiscal Theory of the Price Level*

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Abstract

The fiscal theory of the price level asserts that the price level is determined by the ratio of outstanding public nominal debt into the present value of real primary budget surpluses of the government. We here argue that price determinacy, in general, fails when at least part of the public debt takes the form of securities of infinite maturity. Indeed, price determinacy requires non-Ricardian fiscal plans and a predetermined nominal debt of the government. As no equilibrium restriction prevents the occurrence of a speculative bubble on infinite-maturity public debt, the initial nominal debt of the government is indeterminate and so is the price level under canonical specifications of non-Ricardian fiscal plans.

Keywords: fiscal policy, monetary policy, speculative bubbles, long-term public debt.

JEL Classification Numbers: D50, E40, E50.

1 Introduction

A non-Ricardian fiscal regime is defined by Woodford [16, 18, 19, 20] as a policy such that government primary surpluses or revenues from inflation need not be readjusted after some initial fiscal disturbance. This policy contradicts a common understanding, exemplified by Sargent and Wallace’s [13, 14] unpleasant

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monetarist arithmetic, according to which a fall in money creation can only be followed by a reduction of future government deficits or a rise in future inflation.

In a non-Ricardian regime, the price level is determined by the present value budget constraint of the government. In fact, treating seignorage as negligible, equilibrium requires a sort of valuation equation for government debt of the form

\[
\frac{\text{Nominal government debt}}{\text{Price level}} = \text{Present value of primary surpluses}.
\]

Outstanding nominal debt is predeterminate and, in a non-Ricardian regime, primary public budget surpluses are set according to some given rule, independently of a public intertemporal budget constraint, so that the price level serves to fulfill the valuation equation. In addition, under an interest rate pegging and exogenously given fiscal plans, the most canonical case in the literature, public liabilities, consisting of money balances and government debt, are endogenously determined by the sequence of government budget constraints. This procedure is known as the ‘fiscal theory of the price level’.

In the present note, we argue that price determinacy fails, under a non-Ricardian regime, when primary public surpluses are set exogenously and at least part of the government liabilities are in the form of infinite-maturity nominal securities, like perpetuities, whose market value is to be determined at equilibrium. In fact, we shall show that, as speculative bubbles on infinite-maturity public debt are not ruled out by any of the equilibrium restrictions, the price level is indeterminate under the non-Ricardian regime, when nominal interest is pegged by the Central Bank.\(^1\)

The reason for the indeterminacy is easy to grasp and can be understood through a simple intertemporal accounting. According to the fiscal theory, the price level is jointly determined by the present value of government future surpluses and the current value of government interest bearing bonds. Neglecting seignorage and assuming that public debt entirely consists of perpetuities, at

\(^1\)A similar point is made by Bloise [2] with respect to infinitely-lived real productive assets. Though in both cases speculative bubbles occur at equilibrium, equilibria arising in the case of infinite-maturity public debt differ from those with real productive assets as intertemporal public budget constraint is balanced and, so, speculative bubbles do not require that government liabilities become negative in the long-run, an unappealing feature of equilibria with a speculative bubble on real productive assets.
equilibrium, a valuation equation for government debt imposes

$$\frac{\text{Perpetuity price} \times \text{Perpetuity stock}}{\text{Price level}} = \text{Present value of primary surpluses.}$$

This single restriction involves two distinct unknowns: the price level and the perpetuity price. Illegitimately, assuming that public debt were quoted at its fundamental value,

$$\text{Perpetuity price} = \text{Present value of perpetuity nominal dividends},$$

the valuation equation for government debt would univocally determine the price level.\(^2\) However, there is in fact no equilibrium restriction that can be invoked in order to public debt be quoted at its fundamental value and, in fact, any price of the perpetuity, above its fundamental value, is consistent with equilibrium. Consequently, the price level is indeterminate.\(^3\)

Why are speculative bubbles not ruled out by equilibrium restrictions? Are we omitting a crucial transversality condition? As a matter of fact, consolidating the private sector and the government, infinite-maturity public debt is a nominal security in zero net supply and, in general, speculative bubbles on such a sort of securities are consistent with equilibrium (e.g., Santos and Woodford [12, example 4.3]). In the case of real productive asset, like land, intertemporal accounting (i.e., Walras' Law or, in the case of single representative individual, the intertemporal budget constraint evaluated at market clearing) yields

$$\text{Land price} \times \text{Land stock} = \text{Present value of land stock dividends.}$$

It is such a condition which excludes speculative bubbles, and whose analog, in the case of infinite-maturity public debt, is the valuation equation for the government debt. Hence, there is no further restriction to establish that the perpetuity issued by the government is priced at its fundamental value.

At equilibrium with a speculative bubble, the market value of the government perpetuity might grow unboundedly, but the supply of such a security declines

\(^2\)This is the logic of the fiscal theory of the price level that is presented by Woodford [18] in the case of infinite-maturity public debt, as discussed at pages 19-20 after imposing, without any motivation, restriction (1.19).

\(^3\)Notice that speculative bubbles on government securities of infinite maturity would occur even in a Ricardian policy regime. Also, an infinite maturity of the public debt is essential for this sort of indeterminacy, as speculative bubbles cannot occur on assets of finite maturity under non-arbitrage restrictions.
through time. At equilibrium without a speculative bubble, instead, the supply of the perpetuity might grow unboundedly, though its market price remains stable. If there is inflation at equilibrium (that is, nominal interest exceeds real interest) and fiscal plans for real primary surpluses are set exogenously, the value of the overall intertemporal public revenue grows over time and must be balanced by an increasing value of public debt liabilities.

The fiscal theory of the price level has been mostly developed and advocated by Leeper [8], Sims [15], Woodford [16, 17, 19] and Cochrane [6]. Cochrane [5] has extended the theory to long-term government debt and Dupor [7] has analyzed the consequences on exchange rate determination in an open economy framework. Cochrane [4] and Loyo [9] have argued that the fiscal theory is useful for understanding the actual patterns of inflation in the US (during the seventies) and in Brazil (during the eighties). Namely, a failure of the policymakers, to understand the role of government fiscal policy for price stabilization, may explain why high inflation can arise even without high money creation and limited levels of seignorage. The theory has been also used to show that the price level is well defined even in economies without money (Cochrane [6], Woodford [17]).

The fiscal theory of the price level has been discussed and criticized at length. Buiter [3] objects to the very logic of the theory by arguing that the government must necessarily commit to satisfy an intertemporal budget constraint at all prices (i.e., at equilibrium and out of equilibrium). Treating the government intertemporal budget constraint as an equilibrium restriction leaves one uncertain about the mechanisms responsible for bringing a disequilibrium price level to its equilibrium position. McCallum [10] makes similar claims. Bassetto [1] attempts to clarify the validity of the theory by assuming that the government is a large player and by describing the economy as a game between the government and the private sector. One of the problems with the fiscal theory highlighted by Bassetto [1] is that, in this framework, an unconditional and pre-specified sequence of primary surpluses may not be a valid strategy if the private sector may choose not to lend to the government. Niepelt [11] criticizes the fiscal theory on the ground that, under rational expectations, the value of the outstanding government debt at the beginning of a period cannot be given arbitrarily. Instead, this value must be derived as an equilibrium outcome for the economy in the previous (possibly) unrepresented periods.
None of the above is an objection that we raise in this paper, as we take it for granted that fiscal plans need not satisfy an intertemporal budget constraint at every price sequence and the initial value of government liabilities can be set exogenously.

The contribution most closely related to our paper is Dupor [7], where the use of the fiscal theory as an effective equilibrium selection device is shown to be inadequate. In particular, Dupor finds that an indeterminate price level coexists with a non-Ricardian regime in a two-country economy where governments peg the nominal interest rates on domestic bonds. This type of indeterminacy is connected to the existence of a multiplicity of nominal exchange rates.

The paper is organized as follows. In section 2, we describe a very simple monetary economy with public debt of infinite maturity. In section 3, we present the notion of equilibrium and, in section 4, we show that equilibrium restrictions are consistent with any arbitrary value of speculative bubbles, so delivering an indeterminate price level. Finally, in section 5, we discuss the robustness of this sort of price indeterminacy to other specifications of a non-Ricardian fiscal-monetary regime.

2 A simple economy with money

We shall consider a simple monetary economy with money along the Sidrauski-Brock tradition. Apart from some unessential changes in notation and some simplifying assumptions, the economy is exactly that of Woodford [18] in the case of pure certainty.

There is a continuum of identical individuals, each of which having preferences represented by

\[ \sum_{t=0}^{\infty} \beta^t u \left( c_t, \frac{m_{t+1}}{p_t} \right), \]

where \( c_t \geq 0 \) denotes private consumption in period \( t \) of the single perishable commodity, \( m_{t+1} \geq 0 \) denotes the money balances held by the individual at the end of period \( t \) and \( p_t > 0 \) is the price level in period \( t \) (the price of the single commodity in terms of money). The utility function \( u \) is assumed to be smooth, smoothly strictly concave and smoothly increasing in both arguments. As real balances enter the utility function, it is assumed the existence of liquidity services from money. This allows for non-interest-bearing money to co-exist with
interest-bearing assets.

Apart from money balances, tradable assets consist of a one-period bond and an infinitely-lived security issued by the government as public debt. In every period, the nominal rate of interest is $r_t \geq 0$ and the prices of the infinite-maturity security is $q_t \geq 0$. The security pays off a constant monetary dividend $\rho > 0$ in every period. As short-sales of both assets are allowed, the absence of arbitrage opportunities imposes

$$q_t = \left( \frac{1}{1 + r_t} \right) (q_{t+1} + \rho). \quad (2)$$

In every period, given nominal wealth $w_t$ inherited from previous periods of trade, an individual chooses consumption $c_t \geq 0$, supplies to the market the endowment of the commodity $y_t > 0$ and holds money balances $m_{t+1} \geq 0$, one-period bonds $b_{t+1}$ and public debt $d_{t+1}$ subject to a budget constraint

$$p_t c_t + m_{t+1} + \left( \frac{1}{1 + r_t} \right) b_{t+1} + q_t d_{t+1} \leq w_t + p_t (y_t - s_t), \quad (3)$$

where $s_t > 0$ denotes the real lump-sum tax obligation. Notice that the holdings of both short-term bonds and the infinitely-live security are allowed to be negative. Wealth evolves according to

$$w_{t+1} = m_{t+1} + b_{t+1} + (q_{t+1} + \rho) d_{t+1}. \quad (4)$$

A solvency requirement imposes

$$\frac{1}{a_{t+1}} \sum_{j=1}^{\infty} a_{t+j} p_{t+j} (st+j - yt+j) \leq w_{t+1}, \quad (5)$$

where discount factors are defined by $a_0 = 1$ and

$$a_{t+1} = \left( \frac{1}{1 + r_t} \right) a_t.$$

Finally, the initial wealth is given by

$$w_0 = m_0 + (q_0 + \rho) d_0, \quad (6)$$

where both $d_0 > 0$ and $m_0 \geq 0$ are predeterminate values inherited from the unrepresented past.

Consolidating the sequence of constraints (3)-(6), one obtains an equivalent intertemporal budget constraint of the form

$$\sum_{t=0}^{\infty} \left( \frac{r_t}{1 + r_t} \right) a_t m_{t+1} + \sum_{t=0}^{\infty} a_t p_t (c_t - y_t + s_t) \leq m_0 + (q_0 + \rho) d_0. \quad (7)$$
Asset holdings can be recovered using
\[
\frac{1}{a_t} \sum_{j=0}^{\infty} \left( \frac{r_{t+j}}{1 + r_{t+j}} \right) d_{t+j} m_{t+j+1} + \frac{1}{a_t} \sum_{j=0}^{\infty} a_{t+j} p_{t+j} (c_{t+j} - y_{t+j} + s_{t+j}) = w_t.
\]

This consolidation argument, which relies on the sequential completeness of asset markets, is heavily exploited by Woodford [18, 19].

A complete description of an equilibrium requires a specification of government policy. The central bank sets nominal interest rates \( \{r_t\} \) exogenously and accommodates money balances demand from the private sector at those given interest rates. The government sets fiscal plans \( \{s_t\} \) exogenously and supplies infinite-maturity public debt subject to a sequential budget constraint of the form
\[
m_{t+1} + q_t d_{t+1} = m_t + (q_t + \rho) d_t - p_t s_t,
\]
given that the initial stock of infinite-maturity public debt \( d_0 > 0 \), as well as the initial stock of money balances \( m_0 \geq 0 \), is predeterminate. Notice that the public budget constraint, given all the remaining terms, determines the supply of the infinite-maturity security \( \{d_{t+1}\} \).

3 Equilibrium

Given nominal rates of interest \( \{r_t\} \) and real tax obligations \( \{s_t\} \), an equilibrium consists of a plan \( \{c_t, m_{t+1}, d_{t+1}, b_{t+1}\} \) and prices \( \{p_t, q_t\} \) such that the following conditions are satisfied:

(a) the plan is optimal for the representative individual subject to budget constraint, that is, the plan \( \{c_t, m_{t+1}\} \) maximizes utility (1) subject to the single intertemporal budget constraint (7);

(b) no arbitrage pricing of the infinite-maturity public debt, that is, restriction (2);

(c) sequential public budget constraint, that is, restriction (9), where \( d_0 > 0 \) and \( m_0 \geq 0 \) are predeterminate.

\(^4\)Such an assumption simplifies our argument, which will also apply in the case of an endogenous determination of nominal interest rates through, say, a sort of Taylor rule. Whether the Taylor rule is active or passive (see Leeper [8]) would not be a pertinent issue here. See, for instance, the discussion in Woodford [18, pp. 15-20].
(d) market clearing for commodities and assets, that is,
\begin{align*}
c_t &= y_t \\
b_{t+1} &= 0.
\end{align*}

It should be clear that there are no further equilibrium restrictions than those embodied in conditions (a)-(d). In particular, those conditions imply that the representative individual holds the entire stock of the infinite-maturity security that is supplied by the government. To verify this, use condition (8) to obtain, at equilibrium (that is, under conditions (a)-(d)), the implicit demand of wealth by the representative individual,
\begin{equation}
\frac{1}{a_t} \sum_{j=0}^{\infty} \left( \frac{r_{t+j}}{1 + r_{t+j}} \right) a_{t+j} m_{t+j+1} + \frac{1}{a_t} \sum_{j=0}^{\infty} a_{t+j} p_{t+j} s_{t+j} = w_t. \tag{10}
\end{equation}

This gives a sequence \( \{w_t\} \), beginning with
\begin{equation}
w_0 = m_0 + (q_0 + \rho) d_0, \tag{11}
\end{equation}
because the intertemporal budget constraint (7) holds with the equality (condition (a)). In turn, restriction (10) implies
\[ m_{t+1} + \left( \frac{1}{1 + r_t} \right) (w_{t+1} - m_{t+1}) = w_t - p_t s_t. \]

Assuming that \( w_t = m_t + (q_t + \rho) d_t \) and using the public budget constraint (9), one obtains
\[ \left( \frac{1}{1 + r_t} \right) (w_{t+1} - m_{t+1}) = q_t d_{t+1}. \]

Invoking the no arbitrage condition (2), it follows that
\[ w_{t+1} = m_{t+1} + (q_{t+1} + \rho) d_{t+1}. \]

Because of the initial condition (11), this argument proves market clearing on the asset market by induction.

Notice that (10) can be interpreted as an intertemporal public budget constraint, asserting that outstanding public liabilities, \( w_t \), are backed by revenues accruing from seignorage and tax payments.
The no arbitrage pricing (2) for infinite-maturity public debt, under free-disposal,\(^5\) implies that
\[
\frac{1}{a_t} \sum_{j=1}^{\infty} a_{t+j} \rho \leq q_t.
\]
The left-hand side is the fundamental value of the infinite-maturity security, so that the inequality requires that the market value of the security does not fall below its fundamental value. In fact, Woodford [18, equation (1.19)] imposes
\[
\frac{1}{a_t} \sum_{j=1}^{\infty} a_{t+j} \rho = q_t,
\]
so that infinite-maturity public debt is priced at its fundamental value at equilibrium, and provides no explanation for this. We shall show that restriction (12) is an illegitimate requirement for equilibrium and, as a matter of mere fact, it can be violated at equilibrium.

The notion of equilibrium is a faithful reproduction of that in Woodford [18]. In particular, no solvency condition is imposed on fiscal and monetary plans of the government, so adhering to the non-Ricardian hypothesis.

4 Indeterminacy

We shall here provide a full characterization of equilibrium under simplifying assumptions, none of which is crucial for the argument. To this purpose, assume that the economy is stationary in the endowment \((y_t = y)\) and in interest and fiscal policies \((r_t = r\) and \(s_t = s\), with \(r > 0\)).

At equilibrium, first-order conditions impose
\[
\begin{align*}
p_{t+1} & = \beta (1 + r) p_t, \\m_{t+1} & = p_t \mu,
\end{align*}
\]
where the value of real balances \(\mu > 0\) is obtained so as to satisfy
\[
\frac{\partial u (y, \mu)}{\partial c} = \left( \frac{1 + r}{r} \right) \frac{\partial u (y, \mu)}{\partial \mu}.
\]
In fact, condition (13) corresponds to the intertemporal arbitrage, whereas condition (14) equates marginal utility of real balances to the liquidity cost of
\[^5\text{Which precludes that the price of the security } q_t \text{ be negative; otherwise, our indeterminacy conclusion is even amplified.}\]
holding real balances. The single intertemporal budget constraint reduces to

\[
\left( \frac{1}{1-\beta} \right) \left( \frac{r}{1+r} \right) \mu + \left( \frac{1}{1-\beta} \right) s = m_0 + \frac{(q_0 + \rho) d_0}{p_0}.
\]

(15)

In addition, the no arbitrage condition (2) requires

\[
q_{t+1} = (1 + r) q_t - \rho.
\]

(16)

Finally, sequential public budget constraint (9) can be written (except in the initial period \(t = 0\)) as

\[
d_{t+1} = \frac{1}{q_t} \left( \frac{1 - \beta (1 + r)}{\beta (1 + r)} \right) p_t \mu + (q_t + \rho) d_t - p_t s.
\]

(17)

Equilibrium conditions (a)-(d) reduce to satisfying equations (13)-(17), which account for all restrictions, jointly with non-negativity constraints on prices \(\{p_t\}\), security prices \(\{q_t\}\) and, possibly, security supplies \(\{d_{t+1}\}\). As conditions (13)-(17) are difference equations, with initial given values \(d_0\) and \(m_0\), the determination of equilibrium variables simply requires an initial price \(p_0\) for the commodity and an initial price \(q_0\) for the infinite-maturity security. Differently from the case of a short-term public debt, two unknowns are to be determined by a single restriction (15) corresponding to the intertemporal government budget.

Thought this is illegitimate, it is useful to assume that the infinite-maturity security be priced at its fundamental value in order to understand the determinacy claim of Woodford [18, pp. 19-20]. Condition (12) reduces to

\[
\rho = q_t.
\]

(18)

This, in turn, allows to write restriction (15) as

\[
\left( \frac{1}{1-\beta} \right) \left( \frac{r}{1+r} \right) \mu + \left( \frac{1}{1-\beta} \right) s = m_0 + \frac{(\rho/r + \rho) d_0}{p_0}.
\]

(19)

As a matter of fact, (19) is an equation is the single unknown \(p_0\). If infinite-maturity public debt were quoted at its fundamental value, equilibrium would be fully determined, as claimed by the fiscal theory of the price level.

It should be clear that nothing prevents the infinite-maturity public debt to be priced above its fundamental value at equilibrium. In fact, for some speculative value \(\gamma \geq 0\), assume that

\[
q_t = \frac{\rho}{r} + (1 + r)^t \gamma.
\]
This is consistent with all equilibrium restrictions (13)-(17). To understand this, we shall assume that $\mu = 0$, which corresponds to Woodford’s hypothesis of a limit cash-less economy (and, also, that $m_0 = 0$). Under such additional simplifying assumptions, the sequential public budget constraint (17) reduces to

$$d_{t+1} = \frac{1}{q_t} ((q_t + \rho) d_t - p_t s),$$

which is solved by

$$d_t = \beta^t (1 + r)^t \left( \frac{(\rho/r)(1 + r) + \gamma}{(\rho/r)(1 + r) + (1 + r)^t \gamma} \right) d_0,$$

with

$$p_0 = \frac{(1 - \beta)}{s} \left( \frac{(\rho/r)(1 + r) + \gamma}{} \right) d_0.$$

In the case of no speculative bubble ($\gamma = 0$), the supply of infinite-maturity public debt is exploding whenever $\beta (1 + r) > 1$. On the contrary, when there is a speculative bubble ($\gamma > 0$), the supply of infinite-maturity public debt must be declining.

Notice that, even though there is a speculative bubble, the intertemporal budget constraint is balanced at equilibrium, that is,

$$\frac{1}{a_t} \sum_{j=0}^{\infty} \left( \frac{r_t}{1 + r_t} \right) a_{t+j} m_{t+j+1} + \frac{1}{a_t} \sum_{j=0}^{\infty} a_{t+j} p_{t+j} s_{t+j} = m_t + (q_t + \rho) d_t.$$

The entire public liability is backed by the present value of real primary surpluses, plus possibly revenues from seignorage.

## 5 Remarks

Our elementary analysis shows that, if the Central Bank pegs nominal interest, the price level is indeterminate when at least part of public liabilities consists of infinite-maturity securities, even under a non-Ricardian policy regime. We shall here briefly discuss robustness of this sort of price indeterminacy.

As the equilibrium without speculative bubble is locally isolated, it could be selected on this ground. However, it is to be noticed that any other equilibrium with positive speculative bubble is also locally isolated and, thus, that selection criterion is perhaps inadequate. Indeed, in our simplified economy, comparing

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\(^{6}\)We thank an anonymous referee for suggesting this criterion.
two distinct equilibria, the distance between equilibrium prices of the perpetuity becomes arbitrarily large in the long-period, as

$$| q^a_t - q^b_t | = (1 + r)^t | \gamma^a - \gamma^b |.$$  

Woodford [19] proposes a slightly different formulation of the fiscal theory of the price level. When considering the case of an infinite-maturity public debt, he assumes that the government is able to carry out a security price support policy, so in fact pegging the price of the security over time, consistently with a given path of nominal rates of interest. This would, indeed, deliver a full determinacy of the price level. Contrasting Woodford [19] with Woodford [18], which we faithfully reproduce in our analysis, it is to be noticed that a security price support is not a heuristic hypothesis for simplifying the analysis, but a precise claim on the conduct of monetary policy in order to obtain price determinacy. Statements on the determinacy of the price level need be qualified, in particular, when public debt consists of various securities of infinite maturity and nominal interest is pegged by some mechanical Taylor rule.

Finally, other non-Ricardian specifications of fiscal plans could deliver a determinate price level. This requires that fiscal surpluses be increased proportionally with the speculative bubble, so as to neutralize the effect on the price level. In our simplified economy, an example of such bubble-neutralizing fiscal plans would be given by

$$s_t = \begin{cases} 
    s + \gamma (d_0/p_0), & \text{if } t = 0, \\
    s, & \text{if } t > 0.
\end{cases}$$

Substituting this rule into the intertemporal public budget constraint would yield restriction (19), so pegging a unique price level. Notice, however, that these fiscal plans do not prevent the occurrence of speculative bubbles on infinite-maturity public debt, so that there is still a continuum of equilibria. Also, with heterogeneous individuals, speculative bubbles will, in general, result in a redistribution of initial nominal claims among individuals, so bearing effects of allocative relevance, unless the tax burden is distributed across individuals proportionally with their initial holdings of the infinite-maturity public debt. Furthermore, even with a representative individual, under uncertainty, fiscal plans should be very sophisticated in order to neutralize the effects on the variability of inflation rates across states of nature.

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7 We thank an anonymous referee for suggesting this policy.
References


