Problem Set 4
Econometric Theory
Due on March, 15th 2017

1. Consider the model:

\[ y = X\beta + u \]

where \( X \) is a \( n \times k \) matrix of regressors, \( \beta \) is a \( k \times 1 \) vector of parameters to estimate, the dependent variable \( y \) and the error \( u \) are \( n \times 1 \). To estimate \( \beta \) you propose the following estimator

\[ \tilde{\beta} = (X'X + \lambda I_k)^{-1}(X'y) \]

where \( \lambda > 0 \) is a constant. Assume that \( E[X'u] = 0 \). Is \( \tilde{\beta} \) consistent for \( \beta \)? What is the asymptotic distribution of \( \tilde{\beta} \).

2. Consider the following

\[ y_i = \beta x_i + e_i, \quad i = 1, \ldots, n \]

with \( E[e_i|x_i] = 0 \), where \( x_i \) is here a scalar (and so is \( \beta \)).

In a creative impetous, you best friend proposes, to estimate \( \beta \), the following estimator

\[ \tilde{\beta} = \frac{\sum_{i=1}^{n} x_i^3 y_i}{\sum_{i=1}^{n} x_i^4} \]

(a) Is \( \tilde{\beta} \) unbiased for \( \beta \)?
(b) Is \( \tilde{\beta} \) consistent for \( \beta \)?
(c) Derive the asymptotic distribution of \( \sqrt{n}(\tilde{\beta} - \beta) \).
(d) Find the conditional variance of \( \tilde{\beta} \), given \((x_1, \ldots, x_n)\), assuming homoskedasticity, i.e. \( E[e_i^2|x_i] = \sigma^2 \)?
Clearly state the additional assumptions you may need to answer the previous questions.

3. Show that, if the vector \( x_i \) contains a constant term, and \( E[x_i' u_i] = 0 \), then, for the model

\[
y_i = x_i \beta + e_i
\]

the following properties are true.

(a) \( \bar{e} = 0 \)

(b) \( E[\hat{y}e] = 0 \)

(c) \( \frac{1}{n} \sum_{i=1}^{n} \hat{y}_i = \bar{y} \).