Problem Set 1

Econometric Theory

Due on 22/02/2017

Instructions

• You must work in groups of min 4, max 5 persons.

• The composition of the group cannot change during the entire semester. Changes in composition will be allowed only under very limited circumstances. That means that you should choose your teammates very carefully.

• You should hand back your solved PS to the TA to which the majority of the students are assigned. In case of a tie, toss a coin and choose one of the TA. But remember, all you PS should be handed back to the same TA.

• All problem sets, unless you are otherwise instructed, must be sent as attachment to an email addressed to the TA Wednesday before 9 a.m.

• The subject of the email must be ET2017PSXX, where XX is 01 for Problem Set 1, 02 for Problem Set 2, and so on.

• Clearly list the composition of the group both in the document and in the email. Listing the composition means including the names of all group’s member and their ID number.

• The only format accepted is a Julia Notebook file. Use the same format for the theoretical question using the markdown cells.
Questions

1. [Julia] Generate 10000 normally distributed $N(0,1)$ observations using the command `randn`. Compute the fraction of observed values above 1.96 or below -1.96. How much should this approximately be? (*Hint: A element-wise logical comparison is done by pre-pending . (a dot) to the logical operator.*)

2. The sequence of random vectors $\{X_n\}$ is such that $X_n \xrightarrow{d} X$ where $X \sim N(0, I_k)$. To what random variable the sequence $Z_n = X_n'X_n$ converge in distribution to?

3. Let $X \sim N(0,1)$ and $Y = X^2$.
   
   (a) Show that $E[Y|X] = X^2$.
   (b) Show that $E[Y] = 1$.
   (c) Show that $E[XY] = 0$.

4. Let $X$ and $Y$ two random variables with $E[X] = \mu_X$ and $E[|Y|] < \infty$. Prove that $E[Y|X] = c$, where $c$ is a constant, implies (a) $E(XY) = \mu_X c$; and (b) $Cov(X,Y) = 0$.

5. Let $u$ and $x$ be random variables. Suppose you know that $E[u^2] < \infty$. Give (motivating it) a condition on the moments of $x$ that ensures that

   $$E[ux^2] < \infty.$$ 

6. Let $\{u_i\}_{i \leq n}$ a sequence of i.i.d. random variables and $\{X_i\}_{i \leq n}$ a sequence of random vectors taking values in $\mathbb{R}^k$. Show that for each $i \leq n$ the following equality holds

   $$E[u_i|X_1, \ldots, X_n] = E[u_i|X_i].$$

   (*Hint: Notice that $f(u_i|X_1, \ldots, X_n) = f(u_i, X_1, \ldots, X_n)/f(X_1, \ldots, X_n).$*)