Price Stability with Imperfect Financial Integration

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Abstract

This paper studies whether the international monetary system can be affected by asymmetries in the cross-country positions in the international financial markets, i.e. the fact that some countries are large debtors while others are creditors. An important channel that is explored is the interaction between international risk sharing and the stabilization role of monetary policy in each country. The main finding is that the welfare costs of incomplete markets and the gains of deviating from a policy of price stability are increasing with the cross-country asymmetries in the initial net international positions and in particular they become non-negligible when the steady-state asset position increases above 30% of GDP. When global imbalances increase, optimal monetary policy requires an increase in the volatilities of the asset real returns and in particular of the nominal interest rates which should be also more correlated across countries.
1 Introduction

The last fifteen years have recorded sizeable and unprecedented current account deficits run by the United States accompanied by a gradual deterioration of the U.S. net international investment position that reached the 25% of GDP in the year 2005. Almost three-quarters of the world’s surpluses are absorbed by the U.S. deficit. Projections at current conditions show that levels of net external liabilities around 50% of GDP are not far from being reached in a ten-year horizon. As documented by Lane and Milesi-Ferretti (2001), these developments have been paralleled by an increase in international financial diversification through instruments of different risk and liquidity characteristics. For the U.S. both assets and liabilities have increased up to 75% and 100% of GDP. These developments are usually welcomed for the gains that arise because of more integrated financial markets. Still, net negative positions in the international markets matter and global imbalances might have important negative macroeconomic consequences.

This paper studies whether the international monetary system can be affected by the presence of large asymmetries in the positions in international financial markets, i.e. the fact that some countries are large debtors while others are creditors. An important channel that will be explored is the interaction between international risk sharing and the stabilization role of monetary policy. In an important paper Obstfeld and Rogoff (2002) have shown that in a distorted economy with lack of full international risk-sharing self-oriented policies that achieve price stability in each country can replicate the cooperative outcome. The spillovers that monetary policymakers have on the risk-sharing margin are of second-order importance. This paper re-address this issue in a two-country dynamic model that solves for the optimal cooperative monetary policy when countries have non-zero, but specular, positions in the international financial markets.

The understanding of whether there should be deviations from a policy of price-stability at the international level goes parallel with the analysis of the costs of market incompleteness. The main finding is that the welfare costs of incomplete markets are increasing with the cross-country asymmetries in the initial net international positions and in particular they become non-negligible when the steady-state asset position increases above 30% of GDP. In these cases there are also important gains of deviating
from a policy of price stability. Whereas optimal monetary policy requires a modest increase in the volatility of the producer-price inflation rates, the important adjustment should come through an increase in the volatility of the asset real returns. This is mostly reflected in an increase in the volatility of the nominal interest rates in both countries. Indeed, appropriate movements in the cross-country returns of the financial positions can correct asymmetries in the business cycle synchronization improving risk sharing. Moreover, optimal monetary policy – in the calibrated example – requires more synchronization of the cross-country nominal interest rates as global imbalances become larger. A policy of price stability instead commands a mildly negative correlation independent of the size of the global imbalances.

The paper first presents the zero initial asset holdings case followed by the general one. Thus, once the simple version of the incomplete-market model is understood, we can then point out the additional critical aspects driving the more general case. The detailed structure of the work is as follows. Section 2 presents the model. Section 3 discusses the incomplete-market equilibrium versus the complete one. Section 4 presents the welfare criterion. Section 5 computes the welfare costs of incomplete markets under a policy of price stability and the gains obtained by pursuing the optimal cooperative policy, assuming zero initial foreign asset holdings. Section 6 analyzes, instead, the more general case in which the initial holdings of foreign assets are different from zero. Finally, Section 7 concludes.

2 The model

The model belongs to the class of stochastic general equilibrium models that have been used for the evaluation of monetary policy both in the closed and open-economy literature. The important addition is the treatment of an incomplete-market asset structure that can be directly compared to the complete-market one used in the literature.\(^1\) Our utility-based welfare criterion also allows for a direct evaluation of the welfare costs of imperfect risk-sharing, with particular emphasis on the different

assumptions on the structural parameters of the model, the nature of the shocks—whether supply or demand—and the role of monetary policy.2

We consider a world with two countries, $H$ and $F$. The population on the segment $[0, n]$ belongs to country $H$ while the population on the segment $(n, 1]$ belongs to country $F$. In each country, a continuum of differentiated goods is produced with measure equal to the population size. The utility of a generic consumer $j$ belonging to country $H$ is

$$U^j = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_j^{1-\rho} g_t}{1-\rho} - \frac{1}{n} \int_0^n y_j^i (h)^{1+\eta} z_t^{-(1+\eta)} \right] \right\}$$

where $E_0$ denotes the expectation conditional on the information set at date 0, while $\beta$ is the intertemporal discount factor, with $0 < \beta < 1$.

Households enjoy utility from goods consumption, while they receive disutility from producing goods. The utility function is separable in these two factors. Moreover, each household contributes to the productions of all the goods produced in their own country with a separable disutility.3 $g$ is a preference shock, whereas $z$ is a productivity shock. These shocks are country specific. With starred variables we denote country’s $F$ variables.

The consumption index $C^j$ is defined as follows:

$$C^j = \left[ n^\frac{1}{\sigma} (C^j_H)^{\frac{\sigma - 1}{\sigma}} + (1 - n)^{\frac{1}{\sigma}} (C^j_F)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \left[ \right]$$

where $C^j_H$ and $C^j_F$ are indexes of consumption across the continuum of differentiated goods produced respectively in country $H$ and $F$,

$$C^j_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c^j (h)^{\frac{\sigma - 1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma - 1}}, \quad C^j_F = \left[ \left( \frac{1}{1 - n} \right)^{\frac{1}{\sigma}} \int_1^n c^j (f)^{\frac{\sigma - 1}{\sigma}} df \right]^{\frac{\sigma}{\sigma - 1}}.$$


3This form of utility function can be seen as the outcome of a decentralized labor market in which households get disutility from supplying hours across all the firms within a country. This disutility is separable in the various efforts provided. On the other side, firms employ work, which is perfectly substitutable in production, from all the households belonging to their country.
The elasticity of substitution across goods produced within a country is denoted by \( \sigma \), which is assumed greater than one, while the elasticity of substitution between the bundles \( C_H \) and \( C_F \) is \( \theta \).

We assume that all the goods are traded and that the law of one price holds. We further assume that the same composition of the consumption bundle \( C \) applies to country \( F \). Given these assumptions, it follows that purchasing power parity holds, i.e. \( P = S P^* \), \( P_H = S P^*_H \) and \( P_F = S P^*_F \), where \( S \) is the nominal exchange rate. Here we define the relative price \( T \), the terms of trade, as \( T \equiv P_F/P_H \).

The household \( j \)'s demand of a generic good \( h \), produced in country \( H \), and of the generic good \( f \), produced in country \( F \), are

\[
\begin{align*}
c^j(h) &= \left( \frac{p(h)}{P_H} \right)^{-\sigma} \left( \frac{P_H}{P} \right)^{-\theta} C^j, \\
c^j(f) &= \left( \frac{p(f)}{P_F} \right)^{-\sigma} \left( \frac{P_F}{P} \right)^{-\theta} C^j.
\end{align*}
\]

Aggregating across all households in the world economy, we can write total demands of good \( h \) and \( f \) as

\[
\begin{align*}
y^d(h) &= \left( \frac{p(h)}{P_H} \right)^{-\sigma} \left( \frac{P_H}{P} \right)^{-\theta} C^W, \\
y^d(f) &= \left( \frac{p(f)}{P_F} \right)^{-\sigma} \left( \frac{P_F}{P} \right)^{-\theta} C^W
\end{align*}
\]

where world consumption \( C^W \) is defined as

\[
C^W \equiv \int_0^1 C^j dj.
\]

We assume that households that belong to country \( H \) can allocate their wealth among two bonds. Both bonds are risk-free with one-period maturity. One is denominated in domestic currency and the other in foreign currency. In contrast, households that belong to country \( F \) can allocate their wealth only in a risk-free nominal bond denominated in the foreign currency. So, the only asset traded internationally is the foreign bond.\footnote{This paper considers also the specular case in which the only asset traded internationally is the domestic nominal bond.}

Thus, the budget constraint of household \( j \) in country \( H \) (expressed in real terms with respect to the consumption-based price index) is

\[
\frac{A^j_{H,t}}{P_t(1 + i_t)} + \frac{S_t A^j_{F,t}}{P_t(1 + i^*_t)} \frac{1}{\phi \left( \frac{S_t A_{F,t}}{P_t} \right)} \leq A^j_{H,t-1} + S_t A^j_{F,t-1} + (1 - \tau) \frac{1}{P_t} \frac{1}{n} \int_0^n p_t(h) y_t(h) dh - C^j_t.
\]
at each date $t$. $A_{H,t}^j$ is household $j$’s holding of the risk-free one-period nominal bond, denominated in units of currency $H$. The nominal interest rate on this bond is $i_t$. $A_{F,t}^j$ is household $j$’s holding of the risk-free one-period nominal bond, denominated in units of currency $F$. The price of this bond is inversely proportional to its gross nominal interest rate $1 + i^*_t$. The factor of proportionality is the function $\phi(.)$ which depends on the real holdings of the foreign assets in the entire home economy. This means that domestic households take the function $\phi(.)$ as given when deciding on the optimal holdings of the foreign bond. We require some restrictions on $\phi(.)$: $\phi(0) = 1$ and that $\phi(.)$ assumes the value 1 only if $A_{F,t} = 0$; $\phi(.)$ is a, differentiable, (at least) decreasing function in the neighborhood of zero.

The function $\phi(.)$ captures the costs, for the households belonging to country $H$, of undertaking positions in the international asset market. As borrowers, they will be charged a premium on the foreign interest rate; as lenders, they will receive a remuneration lower than the foreign interest rate. Another way to describe this cost is to assume the existence of intermediaries in the foreign asset market (which are owned by the Foreign households) who can borrow from and lend to households of country $F$ at the rate $1 + i^*$, but can borrow from and lend to households of country $H$ at the rate $(1 + i^*) \cdot \phi(.)$. The profits from this activity are given by

$$K = \frac{A_{F,t}}{P_t} \frac{1}{(1 + i^*_t)} \left[ \frac{1}{\phi \left( \frac{A_{F,t}}{P_t} \right)} - 1 \right] > 0$$

which are positive, given the shape of the function $\phi(.)$. For characterizing the incomplete financial structure, we do not really need to introduce this additional cost. However, this will be useful in pinning down a well-defined steady-state for consumption and assets.\footnote{For related use in small open-economy models, see Senhadji (1997), Schmitt-Grohé and Uribe (2000) and Tuladhar (2003). Ghironi (2000) discusses how an overlapping generation structure can succeed in determining a steady-state asset position.}

In characterizing the budget constraint, we also assume that all the households belonging to a country share the revenues from running the firms in equal proportion. Finally, $\tau$ denotes a country-specific proportional tax on nominal income, while $TR^j_t$ denotes the government transfers to household $j$. The budget constraint at date $t$ of
the fiscal authority in country $H$ is

$$\tau \int_0^n p_t(h)y_t(h)dh = \int_0^n TR^j_t.$$ 

Households of country $F$ can transfer wealth across periods by using only the risk-free nominal bond denominated in units of their own currency. However, they do not face any cost of intermediation and they can lend and borrow at the risk-free nominal interest rate $i^*$. Their budget constraint is

$$\frac{A^{ij}_{F,t}}{P_t^*(1 + i^*_t)} \leq \frac{A^{ij}_{F,t-1}}{P_t^*} + (1 - \tau^*) \frac{1}{1 - n} \frac{1}{P_t^*} \int_1^{1-n} p_t^*(f)y_t^*(f)df + \frac{K}{1-n} - C_t^{ij} + \frac{TR_{t}^{ij}}{P_t^*}. $$

We further assume that the initial level of wealth is the same across all the households belonging to the same country. This assumption, combined with the fact that all the households within a country work for all the firms sharing the profits in equal proportion, implies that within a country all the households face the same budget constraint. In their consumption decisions, they will choose the same path of consumption. We can then drop the index $j$ and consider a representative household for each country. However, consumption will not necessarily be risk shared at an international level. Optimal consumption and portfolio choices imply the following Euler equations in country $H$

$$U_C(C_t, \xi_{C,t}) = (1 + i_t) \beta E_t \left\{ U_C(C_{t+1}, \xi_{C,t+1}) \frac{P_t}{P_{t+1}} \right\}, \quad (5)$$

which reflects trading in the bond denominated in domestic currency and

$$U_C(C_t, \xi_{C,t}) = (1 + i^*_t) \phi \left( \frac{A_{F,t}}{P_t^*} \right) \beta E_t \left\{ U_C(C_{t+1}, \xi_{C,t+1}) \frac{P_t^*}{P_{t+1}^*} \right\}, \quad (6)$$

which reflects trading in the bond denominated in foreign currency. The aggregate budget constraint can be obtained by integrating the budget constraints of the households belonging to the economy together with that of the government as follows

$$\frac{A_{F,t}}{P_t^*(1 + i^*_t)} \phi \left( \frac{A_{F,t}}{P_t^*} \right) = \frac{A_{F,t-1}}{P_t^*} + \left( \frac{P_{H,t}}{P_t} \right)^{1-\theta} C_t^W - C_t. \quad (7)$$

To derive (7), we have imposed the condition that bonds denominated in currency $H$ are in zero-net supply within that country. For country $F$ there is just one Euler
equation
\[ U_C(C_t^*, \xi_{C,t}^*) = (1 + i_t^*) \beta E_t \left\{ U_C(C_{t+1}^*, \xi_{C,t+1}^*) \frac{P_{t+1}^*}{P_t} \right\}, \] (8)
which reflects trading in the bond denominated in foreign currency. The resource constraint of country \( F \) is redundant for the analysis that follows by Walras’s law, having assumed that profits of intermediation are added up to the resources of the foreign economy.

2.1 Price-setting decisions

In this model suppliers behave as monopolists in selling their products. They can affect the quantity demanded through their pricing decisions as shown in equation (4). However, they are small with respect to the overall market and take as given the indexes \( P, P_H, P_F \) and \( C, C^* \). Prices are subject to changes at random intervals as in the Calvo-Yun model.\(^6\) In each period, a seller faces a fixed probability \( 1 - \alpha \) of adjusting the price, irrespective on how long it has been since the last change had occurred. In this event the price is chosen to maximize the expected discounted profits under the circumstance that the decision on the price is still maintained; in fact, the seller also assumes that the price chosen at a certain date \( t \) will apply in the future at date \( t + k \) with probability \( \alpha^k \). It is important to note that all the sellers that belong to the same country and that can modify their price at a certain time will face the same discounted future demands and marginal costs under the hypothesis that the new price is maintained. Hence they will set the same price. We denote with \( \tilde{p}_t(h) \) the price of the good \( h \), in country \( H \), chosen at date \( t \) and with \( \tilde{y}_{t,t+k}(h) \) the total demand of good \( h \) at time \( t+k \) under the circumstances that the price \( \tilde{p}_t(h) \) still applies. From (4), \( \tilde{y}_{t,t+k}(h) \) is

\[ \tilde{y}_{t,t+k}(h) = \left( \frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left( \frac{P_{H,t+k}}{P_t} \right)^{-\theta} C_{t+k}. \]

The optimal choice of \( \tilde{p}_t(h) \) is:

\[ \tilde{p}_t(h) = \frac{\sigma}{(\sigma - 1)(1 - \tau)} \frac{E_t \sum_{k=0}^{\infty} (\alpha \beta)^k \tilde{y}_{t,t+k}(h) z_{t+k}^{-\eta} \tilde{y}_{t,t+k}(h)}{E_t \sum_{k=0}^{\infty} (\alpha \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(h)}, \] (9)

where $\lambda$ is the marginal utility of nominal income which is common across agents within a country. To simplify the analysis, we assume that the distorting taxes are set in a way to offset the monopolistic distortions, i.e. $(1 - \tau) = \sigma / (\sigma - 1)$. Under the Calvo-style price-setting behavior, a fraction $(1 - \alpha)$ of sellers, that can choose to adjust the price, sets the same price. Thus, we obtain the following state equation for $P_{H,t}$

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha) \tilde{p}_t(h)^{1-\sigma}. \quad (10)$$

A similar optimal price-setting decision also holds in country $F$, with the appropriate starred variables.

### 2.2 Complete asset structure

The incomplete asset market model of the previous section is compared to a complete-market one. In this case, both domestic and foreign households can trade in a set of state-contingent securities that deliver one unit of the home and/or foreign currency in each state of nature. Under this market structure, the marginal utilities of consumption will be proportional and can be equated at all dates and states of nature by an appropriate choice of the initial asset allocation

$$C_t^{-\rho} g_t = C_t^{*-\rho} g_t^*. \quad (11)$$

In characterizing the allocation of consumption, conditions (5) and (8) still hold. Instead, conditions (6) and (7) are not anymore relevant. In fact, under complete markets, there is no need to outline the path of the current account for characterizing the consumption allocations in both countries. No-Ponzi games and transversality conditions should be appropriately modified.

### 3 Complete versus incomplete asset market equilibria

A good benchmark for the comparison of the equilibrium allocation under sticky prices is the flexible-price complete-market equilibrium. With flexible prices, the
real marginal costs are constant across time and states of nature. In fact, from the price-setting decisions, it follows that

$$C_t - \rho_t g_t \frac{P_{H,t}}{P_t} = \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_t} (C^W_t)^{\eta_t} z_t^{-(1+\eta_t)}$$

(12)

$$C^* - \rho_t g^* \frac{P_{F,t}}{P_t} = \left( \frac{P_{F,t}}{P_t} \right)^{-\theta_t} (C^W_t)^{\eta_t} z_t^{-(1+\eta_t)}.$$  

(13)

Combining equations (11), (12) and (13), one can characterize the allocation of consumptions and relative prices. In particular, using a log-linear approximation around the steady-state (defined in appendix A), we obtain7

$$\tilde{C}^R_t = \rho^{-1} \hat{g}^R_t,$$

$$\tilde{C}^W_t = \frac{1}{\rho + \eta} \hat{g}^W_t + \frac{1 + \eta}{\rho + \eta} \hat{z}^W_t,$$

$$\tilde{T}_t = \frac{1 + \eta}{1 + \theta \eta} \hat{z}^R_t,$$

where a variable with an upper index $W$ denotes a weighted average of the home and foreign variables with weights $n$ and $1 - n$ respectively, while a variable with an upper index $R$ denotes the difference between the home and foreign variables. Thus, $C^W_t$ denotes world consumption and $C^R_t$ denotes the difference between the home and foreign consumptions. Hat variables denote log deviations with respect to the steady state, while tilde variables denote as well log deviations with respect to the steady state but when prices are fully flexible and markets are complete.

The presence of complete markets, as specified, does not necessarily equalize consumption across countries. Only the marginal utilities of nominal and real income are equal across countries.8 Asymmetric demand shocks, that originate from shifts in consumption preferences, create a departure from complete equalization of consumption across countries. World consumption depends only on world shocks. Favorable supply shocks increase world consumption. On contrast, the terms of trade are not affected by demand shocks, since relative consumption adjusts to absorb these shocks.

7 Note that the steady-state around which we log-linearize is the same independent of the market structure.

8 With imperfect pass-through or pricing-to-market, it is the case that only the marginal utilities of nominal income are equated.
Table 1: The sticky-price complete-market model

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho E_t(c_{t+1}^W - c_t^W) = n\hat{i}_t + (1-n)\hat{i}_t^* - \tilde{R}<em>t^W - E_t\pi</em>{t+1}^W$</td>
<td>IS$_W$</td>
</tr>
<tr>
<td>$c_t^R = 0$</td>
<td>IS$_R$</td>
</tr>
<tr>
<td>$E_t\Delta S_{t+1} = \hat{i}_t - \hat{i}_t^*$</td>
<td>UIP</td>
</tr>
<tr>
<td>$\pi_{H,t} = k[(\rho + \eta)c_t^W + (1-n)(1 + \eta\theta)(\tilde{T}_t - \hat{T}<em>t)] + \beta E_t\pi</em>{H,t+1}$</td>
<td>AS$_H$</td>
</tr>
<tr>
<td>$\pi_{F,t}^* = k^<em>[(\rho + \eta)c_t^W - n(1 + \eta\theta)(\tilde{T}_t - \hat{T}<em>t)] + \beta E_t\pi</em>{F,t+1}^</em>$</td>
<td>AS$_F$</td>
</tr>
<tr>
<td>$\tilde{T}<em>{t-1} = \Delta S_t + \pi</em>{F,t}^* - \pi_{H,t}$</td>
<td>TT</td>
</tr>
</tbody>
</table>

Notes: The index $R$ refers to the difference between Home and Foreign variables. The index $W$ refers to a weighted average of Home and Foreign variables with weights $n$ and $1-n$. We have defined the consumption gap as the difference between the consumption that arises under sticky prices and the one that arises under the flexible-price complete-market allocation, i.e. $c_t \equiv \hat{C}_t - \hat{\bar{C}}_t$. While the world natural rate of interest is defined as $\tilde{R}_t^W \equiv \rho E_t\{(\hat{C}_{t+1}^W - \hat{C}_t^W) - (\hat{g}_{t+1}^W - \hat{g}_t^W)}$.

Terms of trade depend only on asymmetric supply shocks. Whenever there are asymmetric disturbances that induce the households in a country to work more, changes in the terms of trade optimally shift part of the burden to the households in the other country.

Tables 1 and 2 summarize the first-order approximation of the sticky-price model under complete and incomplete markets, respectively. The world IS equation, IS$_W$, is derived from a weighted average, with weights $n$ and $1-n$, of the log-linear approximation of equations (5) and (8), which hold under both specifications of the asset structure. As a first step, we define the consumption gap as the difference between the sticky-price allocation, no matter what the structure of the market is, and the flexible-price complete-market allocation, i.e. $c_t \equiv \hat{C}_t - \hat{\bar{C}}_t$. Then, iterating forward the equation IS$_W$, we find that the world consumption gap depends on the present and expected future values of the difference between the world real interest rate and the world natural interest rate, $\tilde{R}_t^W$. The relevant deflator of the world nominal interest rate is the weighted average of the producer inflation rates, $\pi^W$. The ‘relative’ IS equations, respectively IS$_R$ and IS$_R^I$ for the complete and

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$^9$We denote with $\hat{C}_t \equiv \ln C_t/C, \hat{C}_t^* \equiv \ln C_t^*/C, \hat{T}_t \equiv \ln T_t/T, \pi_{H,t} \equiv \ln P_{H,t}/P_{H,t-1}, \pi_{F,t}^* \equiv \ln P_{F,t}^*/P_{F,t-1}^*, \Delta S_t \equiv \ln S_t/S_{t-1}$ and $a_{F,t} \equiv (A_{F,t}/P_t^*) \cdot \hat{\bar{Y}}^{-1}$.
Table 2: The sticky-price incomplete-market model

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( \rho E_t(c_{t+1}^W - c_t^W) = n\hat{t}_t + (1-n)\hat{t}_t^* - \tilde{R}_t^W - E_t\tilde{\pi}_t^W )</td>
<td>IS_W</td>
</tr>
<tr>
<td>( \rho E_t(c_{t+1}^R - c_t^R) = \hat{t}_t - \hat{t}<em>t^* - E_t\Delta S</em>{t+1} )</td>
<td>IS_R</td>
</tr>
<tr>
<td>( E_t\Delta S_{t+1} = \hat{t}_t - \hat{t}<em>t^* + \delta a</em>{F,t} )</td>
<td>UIP^f</td>
</tr>
<tr>
<td>( \beta a_{F,t} = a_{F,t-1} - (1-n)c_t^R - (1-n)\hat{g}_t^R + (1-\theta)(n-1)(\hat{T}_t - \tilde{T}_t) + (1-\theta)(n-1)\tilde{T}_t )</td>
<td>CA^f</td>
</tr>
<tr>
<td>( \pi_{H,t} = k[(\rho + \eta)c_t^W + \rho(1-n)c_t^R + (1-n)(1+\eta)(\hat{T}_t - \tilde{T}<em>t)] + \beta E_t\pi</em>{H,t+1} )</td>
<td>AS_H^f</td>
</tr>
<tr>
<td>( \pi_{F,t}^* = k^*[(\rho + \eta)c_t^W - \rho n c_t^R - n(1+\eta)(\hat{T}_t - \tilde{T}<em>t)] + \beta E_t\pi</em>{F,t+1} )</td>
<td>AS_F^f</td>
</tr>
<tr>
<td>( \hat{T}<em>t = \hat{T}</em>{t-1} + \Delta S_t + \pi_{F,t}^* - \pi_{H,t} )</td>
<td>TT</td>
</tr>
</tbody>
</table>

Notes: The index \( R \) refers to the difference between Home and Foreign variables. The index \( W \) refers to a weighted average of Home and Foreign variables with weights \( n \) and \( 1-n \). We have defined the consumption gap as the difference between the consumption that arises under sticky prices and the one that arises under the flexible-price complete-market allocation, i.e. \( c_t \equiv \tilde{C}_t - \hat{C}_t \). While the world natural rate of interest is defined as \( \tilde{R}_t^W \equiv E_t\{\rho(c_{t+1}^W - \tilde{C}_t^W) - (\hat{g}_{t+1}^W - \tilde{g}_t^W)\} \).

Other definitions are \( k \equiv [(1-\alpha\beta)(1-\alpha)/\alpha] \cdot [1/(1+\eta)] \) and \( k^* \equiv [(1-\alpha^*\beta)(1-\alpha^*)/\alpha^*] \cdot [1/(1+\eta)] \), while \( \delta \equiv -\phi(0)\tilde{Y} \).

incomplete-market model, depend instead on the structure of the asset market. Under complete asset markets, no matter whether prices are sticky or not, equation (11) holds, and the consumption differential purely reflects asymmetric demand shocks. Relative consumption gap will be always equal to zero. Moreover, taking the difference between the log-linear approximation of equations (5) and (8), one observes that uncovered interest parity holds so that a gap between the cross-country nominal interest rates reflects expectations of depreciation in the nominal exchange rate (UIP equation). However, under incomplete asset markets with transaction frictions, uncovered interest parity does not hold and the spread between the nominal interest rates reflects also a premium on top of the expected exchange rate depreciation (UIP^f equation)

\[
\hat{t}_t - \hat{t}_t^* - E_t\Delta S_{t+1} = -\delta a_{F,t},
\]

(14)

If the domestic country is a net lender, i.e. \( a_{F,t} > 0 \), (net borrower, \( a_{F,t} < 0 \)) in the international markets, the expected return on the domestic asset will be negative (positive), since \( \delta > 0 \). Equation (14) is derived by taking the difference between the
log-linear approximation of equations (5) and (6). Instead, under the case that the
domestic nominal bond is the only asset traded internationally – with the foreign
economy bearing the costs of the transaction frictions– equation (14) would still hold
after having substituted $a_{H,t}$ for $a_{F,t}$. As it will be shown later in the section, the
requirement that $\delta$ is non-negative is also needed for the stability of the equilibrium
dynamic. Another implication of equation (14) is that the cross-country real interest
rate differential reflects the expected real exchange rate depreciation and a premium
related to the net foreign asset position. This is consistent with the estimates of
Lane and Milesi-Ferretti (2002) and with what would be also predicted by a portfolio
balance model. An additional characterization of the incomplete market model is
that the consumption gaps are not necessarily equalized across countries. In fact,
combining equations (5) and (8), it can be shown that

$$\rho E_t(c_{t+1}^R - c_t^R) = \hat{\delta}_t - \hat{\delta}_t^* - E_t \Delta S_{t+1},$$

which is the $IS_{R}^I$. Equivalently, by using equation (14) one finds that the relative
consumption gap can be related to the foreign assets holdings

$$\rho E_t(c_{t+1}^R - c_t^R) = -\delta a_{F,t}. \quad (15)$$

When the domestic economy is a net lender (borrower) in the international markets,
its consumption growth is expected to fall (rise) relative to the other country.

Finally, another difference between the two market structures is that with incom-
plete markets the dynamic of the current account (equation $CA^I$) is needed in order
to determine the equilibrium allocation of the model.

The aggregate supply block does not differ across asset structures. As it is common
with a Calvo-style price-setting model, the producer inflation rates depend on the
current and expected discounted future deviations of the real marginal costs from the
steady-state value. However, the specification of the real marginal costs depends on
the market structure. An important difference, under incomplete markets, is that an
increase in the consumption gap of the home country relative to that of the foreign
country pushes home inflation up. In fact, the marginal utility of nominal income
decreases in the home economy and producer prices tend to rise so as to protect
revenues. Aside from this channel, all other elements in the decomposition of the real marginal costs are similar across asset structures. A positive world consumption gap increases both home and foreign inflation, while the difference between the terms of trade under sticky prices and under the complete-market flexible-price allocation creates a dispersion of inflation across countries.

Finally $TT$ denotes the terms of trade identity which shows that the terms of trade growth depends on the exchange rate depreciation and the inflation rate differential. The model is then closed with the policy rules chosen by the two central banks.

Although the complete and incomplete market allocations look different, there is one particular case in which the two allocations coincide independent of the policy followed by the monetary policymakers. This happens when the home and foreign demand shocks are symmetric such that $\hat{g}_t = \hat{g}^*_t$ at all dates $t$, and the intratemporal elasticity of substitution between Home and Foreign goods $\theta$ is unitary, given the initial condition $a_{F,-1} = 0$.

In fact, under such conditions, the current account equation boils down to

$$a_{F,t} = \frac{a_{F,t-1}}{\beta} - (1 - n)c^R_t,$$

which can be combined with equation (15) in a dynamic system of the form,

$$E_t x_{t+1} = V x_t,$$

where $x_t' = [c_t, a_{F,t-1}]$ and $V$ is an appropriate matrix of parameters. Given that $a_F$ is a predetermined variable, there exists a unique and bounded rational expectation solution if and only if there is one eigenvalue inside the unit circle. For this to be the case, $\delta$ should be positive. This happens to be true given the assumption that the function $\phi(,)$ is decreasing in a neighborhood of zero. It can be further shown that in this unique solution

$$a_{F,t} = \mu_1 a_{F,t-1}$$

where $\mu_1$ is the stable eigenvalue of the matrix $M$. It then follows that given the initial condition, $a_{F,t-1} = 0$, $a_F$ is always zero at all dates $t$.

It is a well-established result that, with Cobb-Douglas preferences, the terms of trade provides a risk-sharing role. Indeed this finding has been well emphasized
by Cole and Obstfeld (1991), Corsetti and Pesenti (2001). However, this is true only when there are productivity shocks. In the case of demand shocks, complete risk sharing requires relative consumption to move in the same direction as demand shocks. The terms of trade can still be a vehicle of wealth distribution but this contrasts with its primal role of allocating production efficiently across countries.

4 Welfare criterion

A natural criterion that serves at the same time for the purposes of evaluating the cost of market incompleteness and comparing alternative monetary policy regimes is the sum of the utilities of the consumers

\[ W \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t w_t \right\}, \]

where

\[ w_t \equiv n \frac{C_{t}^{1-\rho} g_t}{1 - \rho} + (1 - n) \frac{C_{t}^{1-\rho} g_t^*}{1 - \rho} - \int_0^n \frac{y_t(h)^{1+\eta} z_t^{-(1+\eta)}}{1 + \eta} dh - \int_n^1 \frac{y_t^*(f)^{1+\eta} z_t^{*-(1+\eta)}}{1 + \eta} df \]

Appendix B shows that a second-order approximation of \( W \), around the steady state in which a taxation subsidy completely offsets the monopolistic distortions in both countries, delivers

\[ W = \frac{-\bar{C}^{1-\rho}}{2} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}, \]

where

\[ L_t = (\rho + \eta) \cdot (c_t^W)^2 + n(1 - n)\rho \cdot (c_t^R)^2 + n(1 - n)(1 + \eta) \theta \cdot (\bar{T}_t - \tilde{T}_t)^2 + n^2 \frac{\sigma}{k} \cdot (\pi_{H,t})^2 + (1 - n) \frac{\sigma}{k^*} \cdot (\pi_{F,t}^*)^2 + \text{t.i.p.} + O(||\xi||^3), \]

and t.i.p. denotes elements that are independent of policy while \( O(||\xi||^3) \) measures residuals of third-order in the maximum amplitude of the shocks.

Using equation (16), one can evaluate the deadweight losses implied by the distortions existing in the model. Once the monopolistic distortions are offset by appropriate taxation subsidies, the flexible-price complete-market allocation is the efficient allocation for the whole economy. Any departure from this allocation produces losses.
for society. Price stickiness is a source of distortions when, combined with staggered prices, creates dispersion of demand across goods that are produced according to the same technology. The squares of the producer inflation rates in each country capture these distortionary costs. On the other side, relative prices should move when there are asymmetric productivity shocks. In fact, the terms of trade should offset asymmetric supply shocks. In the welfare function, this is captured by the square of the terms of trade with respect to their efficient level. Finally, the world and relative consumption gap should be completely stabilized. In particular, a departure from the complete risk-sharing of the marginal utilities of nominal incomes creates welfare costs. And the microfounded welfare criterion delivers appropriate weights for each of these distortions.

It is important to note that when markets are complete and prices are sticky, a policy of zero producer inflation rates in both countries achieves the optimum for society and closes all the gaps in the above loss function.

5 The welfare costs of imperfect risk-sharing and the role of international monetary policy cooperation

There are many angles through which the welfare costs of incomplete markets can be studied in this work since there are many possible monetary regimes that depend on the monetary policy rules followed by each country. A natural candidate is the allocation that corresponds to a policy of zero producer inflation rates in both countries, i.e. $\pi_{H,t} = \pi_{F,t} = 0$ at all dates $t$— a “price stability” policy. First of all, as discussed in the previous section, this combination of policies achieves the first best when markets are complete. Moreover, it implements the flexible-price allocation independently of the asset structure. In fact, the literature on the costs of incomplete markets has mainly focused on real fluctuations. Finally, many studies in the optimal monetary policy literature in closed economy have analyzed the conditions under which strict
price stability is an optimal policy.\textsuperscript{10} A policy of zero producer inflation rate is the natural open-economy counterpart of those analyses.\textsuperscript{11}

After having evaluated the welfare costs of incomplete markets under price stability, this work will move to the analysis of the conditions under which a departure from price stability can substantially reduce these costs.

Let us turn to the implications for the equilibrium allocation of a combined policy of price stability. First it is easy to see that the world consumption gap is zero and that the terms of trade gap moves in the opposite direction of the relative consumption gap as shown by the following equation

\[ \hat{T}_t - \hat{T}_t = \frac{\rho}{1 + \eta \theta} c_t^R. \]  

(17)

If consumption increases in one country relative to the other, producer prices tend to increase in order to offset the fall in the marginal utility of revenue. As a result, the terms of trade of the country with high consumption worsens in order to reduce the pressure to its producer inflation rate. By using condition (17) into \( CA^I \) and equation (15), we can reduce the equations describing the flexible-price incomplete market allocation into a dynamic system as follows

\[ E_t x_{t+1} = D x_t + M \varepsilon_t \]

where \( x'_t = [c_t^R \ a_{F,t-1}] \), \( \varepsilon_t = [\hat{z}_t^R \ \hat{g}_t^R] \), while \( D \) and \( M \) are 2 \times 2 matrices. Assuming generic AR(1) processes for the shocks \( \hat{z}_t^R \) and \( \hat{g}_t^R \), under the conditions of determinacy, the unique and stable solution can be written as \textsuperscript{12}

\[ e_t^R = -e_1 a_{F,t-1} - v_1 (1 - \theta) \hat{z}_t^R - v_2 \hat{g}_t^R, \]

\[ b_t = \mu_1 a_{F,t-1} - v_3 (1 - \theta) \hat{z}_t^R - v_4 \hat{g}_t^R, \]

\textsuperscript{10}See Woodford (2003) for an overview.

\textsuperscript{11}See Benigno and Benigno (2003, 2006) and Sutherland (2001) for a general analysis of the conditions under which this definition of price stability is optimal even from a non-cooperative perspective.

\textsuperscript{12}For determinacy of the equilibrium, it is required that \( \theta \geq \max(0, \vartheta) \) where \( \vartheta = (\rho - 1)/(\eta + \rho) \). Note that if \( \rho \) is less than 1, then \( \vartheta \) is less than 0, while as \( \rho \) increases above 1, \( \vartheta \) becomes positive but always less than 1.
where $\mu_1$ (with $0 < \mu_1 < 1$) is the stable eigenvalue of the matrix $D$, and $e_1$ (with $e_1 < 0$) is the second element of the left eigenvector associated with the unstable eigenvalue of the matrix $D$; $\nu_1$, $\nu_2$, $\nu_3$ and $\nu_4$ are all positive coefficients which are combinations of the structural parameters of the model. Asymmetric demand shocks have an unambiguous effect on the relative consumption gap and the accumulation of assets. As asset markets depart from perfect completeness, it is no longer possible to perfectly insure the marginal utilities of nominal income across countries. When a demand shock happens in country H, consumption rises relative to the other country through an increase in international borrowing; but consumption cannot rise enough to completely match the shock. Hence, the consumption gap of country H falls relative to the other country.

Instead, an asymmetric supply shock, such as a terms of trade shock, has an ambiguous effect on the relative consumption gap and the accumulation of assets. The effect depends on whether the value of the intratemporal elasticity of substitution between Home and Foreign goods is above or below one. When the value of $\theta$ is below unity, a terms of trade shock can result in what is known in the trade literature as ‘immiserising growth’. As shown in Bhagwati (1956), the deterioration of the terms of trade can offset the beneficial effect of an expansion by reducing real income. Indeed, when $\theta$ is less than 1, a temporary positive productivity shock in country $H$ produces a temporary negative effect on the real income of country $H$, consumption decreases less than proportionally because of consumption-smoothing behavior; the level of borrowing increases in order to finance a level of consumption above the fall in the real income. The actual level of the terms of trade overshoots the efficient level. A totally opposite pattern arises in the case when $\theta$ is bigger than 1. A temporary positive productivity shock produces a temporary positive effect on real income, consumption increases, but less than proportionally, and assets accumulate.

Having described the producer-price stability allocation, we move to the quantification of the welfare costs of market incompleteness under the identified price stability policy. The results of the literature are controversial. Some of the papers report very small gains from international risk-sharing (less than 0.1\% of units of steady-state consumption) while others report much higher values (sometimes of the
order of 20%).

Consistent with the literature and following Lucas (1987), we reparametrize the welfare costs in terms of a permanent percentage shift in steady-state consumption. We denote with $W^C$, the welfare under the complete-market producer-price-stability allocation and with $W^I$, the welfare under the incomplete-market producer-price-stability allocation. The permanent percentage shift in steady-state consumption, $\lambda$, can be written as

$$\lambda \equiv \frac{W^C - W^I}{C^{1-\rho}} = \frac{n(1-n)\rho}{2} \cdot \left[ \frac{1 + \theta(\eta + \rho)}{1 + \theta\eta} \right] \cdot E_0 \left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t (c_t^R)^2 \right\}.$$  \hspace{1cm} (18)

It follows that the costs of following a zero-producer-inflation-rate policy with incomplete markets are proportional to a proxy of the variance of the consumption-gap differential. The measure $\lambda$ is highly sensitive to the parametrization used.

Our strategy is to fix the calibration of some parameters and to vary others that are instead more controversial and critical for driving the magnitude of the welfare costs. We set $\beta = 0.99$ which implies that the steady-state real interest rate is around 4% (in a quarterly model). We assume that country $H$ is U.S. while country $F$ is the rest of the world. We assume that countries are of equal size, $n = 0.5$. Following recent empirical works on the estimation of forward-looking aggregate supply equations, we assume that $\alpha$ and $\alpha^*$ are 0.66 and 0.75, which implies that the durations of the contracts are 3 and 4 quarters in U.S. and the rest of the world, respectively. The degree of monopolistic competition is taken from Rotemberg and Woodford (1998), where they set $\sigma = 7.66$ which implies an average mark-up of 15%.

The inverse of the elasticity of producing the goods is calibrated according to Rotemberg and Woodford (1998), $\eta = 0.47$. However, this value implies a Frisch elasticity of labor supply equal to 9.5. Micro-data suggests Frisch elasticity to be in

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14 This is common in the literature on the costs of imperfect risk sharing. A detailed discussion is in Kim et al. (2000) and Van Wincoop (1999).

15 In our model the inverse of the elasticity of producing the goods does not coincide with the inverse of the elasticity of labor supply, since we are not necessarily restricting the analysis to linear production functions.
the range of 0.05 – 0.3. Thus, we also analyze the case \( \eta = 5 \) which corresponds to a Frisch elasticity of 0.28. For the risk-aversion coefficient we choose \( \rho = 1 \), consistently with the work of Eichenbaum et al. (1988) that found a range of 0.5 – 3. Barsky et al. (1987) have instead suggested values greater than 5. We then analyze another scenario in which \( \rho = 6 \). The parameter \( \delta \) measures the cost of intermediation in the market of the bonds denominated in the foreign currency. It can be related to the costs, for the home agents, of borrowing in the foreign currency –a proxy of their default risk. We set \( \delta \) to be \( 10^{-3} \) implying a 10-basis-point spread of the domestic rate (in the foreign-currency market) over the foreign rate. We will also assume \( \delta \) equal to \( 10^{-2} \) and a related spread of 100 basis points.

As outlined in Obstfeld and Rogoff (2000) the intratemporal elasticity of substitution is a critical parameter in this class of open-macro models. According to some recent studies, such as Harrigan (1993) and Treffer and Lai (1999), a sensible assumption for this parameter is 6. RBC literature instead assumes values in the range of 1 – 2. We produce a robustness analysis for values between 0.8 and 6.

Finally, following Heatcote and Perri (2002), we assume the following process for the productivity disturbances

\[
\begin{pmatrix}
\hat{z}_t \\
\hat{z}^*_t
\end{pmatrix} = \begin{bmatrix}
0.97 & 0.025 \\
0.025 & 0.97
\end{bmatrix} \begin{pmatrix}
\hat{z}_{t-1} \\
\hat{z}^*_{t-1}
\end{pmatrix} + \begin{pmatrix}
u_{1,t} \\
u_{2,t}
\end{pmatrix}
\]

where \( u_1 \) and \( u_2 \) are white-noise processes with standard deviations \( \sigma_{u_1} = 0.0073 \) and \( \sigma_{u_2} = 0.0044 \) and correlation equal to 0.29. Since the empirical literature is silent on the estimation of the demand shocks, we assume as it is usually done in the RBC literature (see Stockman and Tesar, 1995) that they are distributed as the productivity shocks.\(^{16}\)

Table 3 shows the welfare costs of incomplete markets under producer-price stability. The top part of the table presents the case of productivity shocks. Consistent with the theoretical results, when \( \theta = 1 \), there are no welfare costs. As \( \theta \) departs from 1, the costs increase in a convex manner. They become higher the higher is the intertemporal elasticity of consumption. But at the end, across the various cases,

\(^{16}\) Between demand and productivity shocks, the white-noise disturbances are statistically independent.
they are quite negligible. At most, they reach 0.01% of a permanent shift in steady-state consumption in the lower-right corner of the table. Table 4 repeats the same experiment in the case of preference shocks. A qualitative difference with respect to the results of Table 3 is that as $\theta$ increases the costs are reduced. But, from a quantitative point of view they are still negligible. It is not a surprise that the gains from deviating from a policy of strict price stability are smaller than 1/1000 of a percent point of a permanent shift in steady-state consumption. These findings are robust across various specifications of the parameters of the model. Thus, we confirm the results of Obstfeld and Rogoff (2002) in a more general model and across various parameter specifications. Self-oriented policies are optimal from a global perspective.

6 Asymmetric holdings of foreign assets

This section is going to show that a critical assumption of previous results is the fact that the net foreign asset position was assumed to be zero in the steady state. It is not a surprise that in such a symmetric environment there are no gains of deviating from a symmetric policy of price stability. However, as documented by Lane and Milesi-Ferretti (2001, 2002), net foreign assets over GDP vary across countries and are in general different from zero. They further argue that the level of net foreign assets is a key state variable and a crucial determinant of the benefits of international financial integration. This is a direction that can be explored here.

More recent literature as Lane and Milesi-Ferretti (2002, 2005) and Tille (2005), has also documented an increased diversification of countries’ portfolio. Accounting for this feature of the data involves a non-trivial modification of the solution method used in this paper. Portfolio diversification requires endogenously-chosen portfolio allocations which can be solved using the method of Devereux and Sutherland (2006). In a log-linear approximation of the structural equations, portfolio choices are going to be reflected in some of the coefficients of the approximation capturing the steady-state portfolio positions. However these coefficients will be function of the ratio of second-order moments and in particular of the optimal policy chosen by the central planner. A linear-quadratic solution method is not appropriate for this problem. This
is an area of open research that goes beyond the scope of this paper.\(^{17}\)

This paper is following the simplest route of assuming that there is only one asset traded internationally either a nominal bond denominated in domestic currency or one in foreign currency. It is likely that the results of this work are going to put an upper bound on the welfare costs of incomplete markets in this class of models. Allowing for greater portfolio diversifications increases risk-sharing lowering further these costs.

To the end of introducing a non-zero steady-state holdings of foreign assets, we need to modify appropriately the function \(\phi(.)\). In particular we assume that \(\phi(.)\) assumes the value of one if and only if \(\frac{A_{t,t}^F}{P_t^*Y_t}\) is equal to \(\bar{a}_F\), which is the steady-state level of foreign assets over GDP held by the domestic country. As before, \(\phi(.)\) is a, differentiable, (at least) decreasing function in the neighborhood of \(\bar{a}_F\). A detailed description of how the log-linear approximations of the structural equations change can be found in Appendix B. Here we focus on the critical novelties. In particular, the current account equation becomes

\[
(1 + \bar{a}_F \delta) \beta w_t = w_{t-1} + \bar{a}_F (\beta \delta - \pi_t^*) + (\theta - 1)(1 - n) \hat{T}_t + \hat{C}_t^W - \frac{s}{n} \hat{C}_t, \tag{19}
\]

where \(s \equiv n(1 + (1 - \beta)\bar{a}_F)\) and \(w_t \equiv A_{t,t}^F/(P_t^*Y_t) - \bar{a}_F\).

To enlighten an important difference, it is now the case that the real return on the asset traded internationally has a first order impact on the country’s revenues and on the current account dynamic. The size of this valuation channel depends on the size of the steady-state asset position. Moreover, monetary policy matters in an important way in influencing the current account dynamics through its direct effect on the foreign asset return. In particular, the real return is influenced by the foreign nominal interest rate together with the foreign CPI inflation rate. In the case instead in which the only asset traded internationally is denominated in the home currency, the current account equation modifies to

\[
(1 + \bar{a}_H \delta) \beta w_t = w_{t-1} + \bar{a}_H (\beta i_t - \pi_t) + (\theta - 1)(1 - n) \hat{T}_t + \hat{C}_t^W - \frac{s}{n} \hat{C}_t,
\]

\(^{17}\)Benigno (2006) performs an analysis of valuation effects in a model with multiple assets traded but in a perfect foresight, so that the steady state portfolio allocation can be taken as given.
where now \( s \equiv n(1 + (1 - \beta)\bar{a}_H) \) and \( w_t \equiv A_{H,t}/(P_t\bar{Y}) - \bar{a}_H \). In contrast to the previous case, it is monetary policy in the home country that matters for this valuation channel. In both cases, monetary policy has now an important distributive and direct role for the dynamics of wealth across countries.

The welfare criterion changes to

\[
W = -\frac{U_C}{2}\mathbb{C}^W - \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\},
\]

with

\[
L_t = (\rho + \eta) \cdot [c_t^W]^2 + s(1-s)\rho \cdot [c_t^R]^2 + n(1-n)(1+\eta \theta) \theta \cdot [\hat{T}_t - \tilde{T}_t]^2 \\
+ n \sigma_k \cdot (\pi_{H,t})^2 + (1-n) \sigma_k^* \cdot (\pi_{F,t})^2 + t.i.p. + \mathcal{O}(\|\xi\|^3),
\]

where the weight given to the risk-sharing costs depends now on the share of home consumption in world consumption.\(^{18}\) To the end of evaluating welfare, we need to calibrate the steady-state foreign asset position. In particular, Lane and Milesi-Ferretti (2005) have shown that there is evidence that the net foreign asset positions in industrial countries are worsening. In particular U.S international net debt position is estimated to be above 20% of the GDP. We set in the benchmark case that \( \bar{a}_F \) or \( \bar{a}_H \) are equal to \(-25\%\) of GDP at an annual rate and we check for worse scenarios in which the steady-state net debt position reaches 50% or 70% of GDP.

Table 6 shows the costs of incomplete markets under the producer-price stability policy following all shocks. The top part of the table considers the case in which the only asset traded is the foreign asset, whereas the bottom part of the table considers the opposite case. Compared with the costs implied by the sum of tables 3 and 4, they are much larger. In the third column, when the steady-state liability position is at 25% of GDP, they are in the range of 0.01-0.02% of a permanent shift in steady-state consumption when the only bond traded is in foreign currency and around 0.01% when the only bond traded is in domestic currency. In general this is true for values of \( \theta \) above 2. They are increasing with the risk aversion coefficient, the inverse of the Frisch elasticity of labor supply and with the magnitude of the transaction costs. Still,

\(^{18}\)Note that the special case of the previous section can be obtained as the limiting case when \( \bar{a}_F = 0 \) or \( \bar{a}_H = 0 \), both in the structural equilibrium conditions and in the welfare criterion.
they are small numbers. Interestingly, a worsening of the net foreign asset position is
the critical factor that increases substantially the magnitude of the costs. They reach
0.04% (0.02%) when the liability position is 50% of GDP and is fully denominated in
foreign (domestic) currency. They increase even to higher numbers when the liability
position reaches 70% of GDP: 0.07-0.08% in one case while spiking to 0.3% in the
other case.

Are there gains from a policy deviating from price stability when the steady-state
net foreign asset position is different from zero? Table 7 investigates this issue. The
gains are very small, lower then 0.01%, when liabilities are in the order of 25% of GDP.
They instead becomes important when the net foreign asset position worsens. When
the overall liabilities reach 50% and 70% of the GDP, a coordinated monetary policy
can reduce substantially the welfare costs of incomplete markets. A producer-price
stability policy is a symmetric policy in an asymmetric world. When the asymmetries
in the initial holdings of foreign assets are important, it cannot succeed in approx-
imating well the first best. Wealth effects induced by asymmetries in the holdings
of foreign assets imply additional welfare costs that can be reduced by appropriate
coordination of monetary policies. But, how does the volatility of macroeconomic
variables change when the net foreign asset position worsens substantially? Figures 1
and 2 compare the standard deviations of relevant macroeconomic variables between
the price stability and the optimal policy when the foreign debt varies from 0% to
100% of GDP and is denominated in foreign currency. The volatility of producer-
price inflation in the home country—the one indebted in the international markets—increases as the liability position worsens. This increase is significant for values of
the debt above 20% of GDP. However, the standard deviation remains contained
towards low numbers, below 0.03% at an annual rate. The volatility of home and
foreign CPI inflation rates and of the nominal exchange rate increases as foreign debt
increases both under the optimal and the price-stability policy. The decrease is more
pronounced under the optimal policy. The most interesting fact is that whereas the
volatilities of the domestic and foreign nominal interest rates are stable under the
price-stability policy, they increase substantially under the optimal policy and again
become important in magnitude when the debt increases above the 20% of GDP.
In particular the volatility of the foreign nominal interest rate increases more than that of the domestic nominal interest rate. Figures 3 and 4 repeat the analysis for the case in which the debt is denominated in domestic currency.\textsuperscript{19} The important difference is that now the volatilities of the nominal exchange rate and of the CPI inflation rates decrease. In a similar way to Figure 2, the volatilities of domestic and foreign nominal interest rates increase substantially, but now it is the volatility of the domestic nominal interest rate that increases by a larger amount. One might be tempted to conclude that the nominal interest rate of the currency in which the international debt is denominated should increase its volatility but more than that of the currency in which there is no debt on the international markets. However, we have seen from equation (19), that what matters for the evolution of the net foreign asset position is the real return. Indeed Figure 5 shows, for the case in which all the debt is denominated in domestic currency, that the volatilities of the real returns in both the domestic and foreign assets increase in a substantial way under the optimal policy while they remain constant under a price-stability policy.\textsuperscript{20}

The intuition for this result depends on the importance of the risk-sharing objective embedded into the welfare function (16). When there are large asymmetries in the initial distribution of wealth, variations in the asset returns can have a large repercussion on the countries’ consumption profiles. If a country receives a positive productivity shock, on a first impact, this affects real income and then consumption. As a consequence, the cross-country consumption differential increases. If on top of that the country is a debtor in the international markets, the impact of the shock on the cost of debt can increase or reduce the amount of financial liabilities of the country and work in the direction of magnifying or reducing the cross-country inflation differentials. In particular, an increase in the financial cost of the outstanding debt worsens the financial position of the country reducing its consumption and enhancing risk sharing. Since an important component of the real return is the nominal interest rate, more volatile nominal interest rate are needed to generate such valuation effects which produce volatile real asset returns. Interestingly, Figure 5 shows that the excess

\textsuperscript{19}In Figures 1-4, the following calibration is used: $\rho = 2$, $\theta = 1.5$, $\eta = 0.47$, $\beta = 0.99$, $\delta = 0.001$, $\alpha = 0.66$, $\alpha^* = 0.75$ where the shocks are distributed as in the previous sections.

\textsuperscript{20}Similar result can be obtained when the debt is denominated in foreign currency.
real return between the two assets is less volatile under the optimal policy than under the price stability policy. Indeed the excess real return captures the deviations from uncovered interest parity and is directly related to the level of foreign assets held by the domestic economy, as shown in equation (14). Enhanced risk-sharing requires reduced volatility of the net-foreign asset position since most of the risk-sharing action occurs through contingent movements in the return of the assets and not necessarily through variations in the financial positions which instead adjust more slowly.

But, is it the case that an increased dispersion in the external financial position of countries requires much more integrated and coordinated monetary policies? Figure 6 answers to this question by showing the correlation of the producer-price inflation rates and of the nominal interest rates under the optimal and the price-stability policy. Under optimal policy both correlations start on the negative side when the steady-state asset position is zero and increase and become positive when the financial position of the countries diverge. Obviously, under a price-stability policy, the correlation of the producer inflation rates is zero, but interestingly the correlation of the nominal interest rates is negative and remains negative no matter the divergence in the financial position of the countries. Furthermore, the last graph shows further that optimal policy requires much more correlated real returns.

7 Conclusion

This paper has shown that the deepness of financial markets associated with large exposures of some countries on the international financial markets is not innocuous for the designing of the international monetary system. This issue becomes more relevant when the asymmetries in the countries’ exposures increase. These results have been obtained in a simple model of incomplete markets in which only one bond is traded and where there are transaction costs in trading in the international markets. However, the increase in financial integration that we observe in the data is accompanied by the proliferation of several financial instruments of different characteristics in terms of risk, liquidity and maturity. Increased global diversification that improves international risk-sharing is able to automatically correct for asynchronized
international business cycles, without the need of monetary policy coordination. But, as Obstfeld (2005) suggests, “the amount of real diversification is surely lower” than what one would expect from a first look at the data because of the many intermediaries through which the financial instruments pass. Under these circumstances, our simplifying assumption can represent a first step towards a more complex analysis.

The market structure assumed in this paper — of incomplete markets combined with transaction costs— has been successfully used to analyze the interaction between supply-side behavior, market structure and the real exchange rate in Benigno and Thoenissen [2003]. For a UK-euro area calibration, they show that when TFP increases in the traded goods sector, a depreciation of the terms of trade offsets the appreciation of the relative price of non-traded goods, contrasting with the Balassa-Samuelson proposition. Moreover Benigno and Thoenissen (2007) has shown that this very simple form of market structures is sufficient in generating the observed cross-correlation between relative consumption and real exchange rate, so to explain the Backus-Smith anomaly. In an empirical analysis, Selaive and Tuesta (2003) find that consumption growth and real exchange rates may be consistent with a significant role for the net foreign asset position as it would be implied by the model of this paper.

An important question that is left to further research is to explain what would happen when each country is just interested in maximizing the welfare of its residents. Indeed, to maximize the consumption of its own residents, each policymaker has an incentive to reduce the financial costs of its liabilities or to increase the return of its assets independently of the business cycle synchronization. This interest contrasts with that of the other policymaker.

References


[31] Selaive, Jorge and Vicente Tuesta [2003], “Net Foreign Assets and Imperfect
Pass-through: The Consumption Real Exchange Rate Anomaly,” FRB Interna-
tional Finance Discussion Paper No. 764

[32] Senhadji, Abdelhak [1997], “Sources of Debt Accumulation in a Small Open

[33] Stockman, Alan C., and Linda Tesar [1995], “Tastes and Technology in a Two-
Country Model of the Business Cycle: Explaining International Comovements,”

[34] Sutherland, Alan [2001], “A Simple Second-Order Solution Method for Dynamic
General Equilibrium Models,” unpublished manuscript, University of St An-
drews.

Rate Fluctuations,” Federal Reserve Bank of New York Staff Report 226, Octo-
ber 2005.

the CES Monopolistic Competition Model,” unpublished manuscript, University
of Toronto.

266-270.

[38] van Wincoop, Eric [1994], “Welfare Gains from International Risksharing,” Journal
of Monetary Economics 34, pp. 175-200.

[39] van Wincoop, Eric [1999], “How Big are Potential Welfare Gains from Interna-

[40] Woodford, Michael [2003], Interest and Prices, Princeton University Press.

[41] Yun, Tack [1996], “Nominal Price Rigidity, Money Supply Endogeneity, and
Appendix A

This appendix solves for the steady-state allocation.

We consider a steady-state in which all the shocks are zero and in which the home and foreign monetary policymakers set their respective CPI inflation rates to zero:

\[ P_t/P_{t-1} = P_t^*/P_{t-1}^* = 1. \]

Equations (5) and (8) then imply that the nominal interest rates, \( 1 + i \) and \( 1 + i^* \) equal to

\[ 1 + i = 1 + i^* = \frac{1}{\beta}, \]

which along with equation (6) implies that \( B_F = 0 \) in the steady state. From equation (7) we obtain that

\[ \bar{C} = \left( \frac{P_H}{P} \right)^{1-\theta} \bar{C}^W, \tag{A.1} \]

and from the resource constraint of the foreign country

\[ \bar{C}^* = \left( \frac{P_F}{P} \right)^{1-\theta} \bar{C}^W, \tag{A.2} \]

where we recall that

\[ n \left( \frac{P_H}{P} \right)^{1-\theta} + (1 - n) \left( \frac{P_F}{P} \right)^{1-\theta} = 1. \tag{A.3} \]

Assuming \( \bar{g} = \bar{g}^* = \bar{a} = \bar{a}^* = 1 \), from the price-setting condition we get

\[ \bar{C}^{-\rho} \frac{P_H}{P} = \left( \frac{P_H}{P} \right)^{-\theta \eta} (\bar{C}^W)^{\eta}, \tag{A.4} \]

\[ \bar{C}^{*-\rho} \frac{P_F}{P} = \left( \frac{P_F}{P} \right)^{-\theta \eta} (\bar{C}^W)^{\eta}. \tag{A.5} \]

Combining conditions (A.1), (A.2), (A.3), (A.4), (A.5), we obtain that \( \bar{C} = \bar{C}^* \) and \( \frac{P_H}{P} = \frac{P_F}{P} = 1. \)
Appendix B

The case with a zero steady-state net foreign-asset holdings.

In this appendix we derive the second-order approximation of equation (17) in the text, following in some steps Woodford (2003). The welfare criterion is

\[ W = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t w_t \right\}, \]

where the utility flow is defined as a weighted average of the utility of both countries, disregarding the liquidity effects of holding real money balance

\[ w_t \equiv n \frac{C_t^{1-\rho} g_t}{1 - \rho} + (1 - n) \frac{C_t^{1-\rho} g_t^*}{1 - \rho} - \int_0^n \frac{y_t(h)^{1+n} z_t^{(1+\eta)}}{1 + \eta} dh - \int_1^n \frac{y_t^*(f)^{1+n} z_t^{(1+\eta)}}{1 + \eta} df. \]  

(B.6)

First, we take a second-order expansion of the term

\[ n \frac{C_t^{1-\rho} g_t}{1 - \rho} + (1 - n) \frac{C_t^{1-\rho} g_t^*}{1 - \rho} \]

around the steady state where \( \bar{C} = \bar{C}^* \) and where \( \bar{g} = \bar{g} = 1 \) at each date \( t \) obtaining

\[ n \frac{C_t^{1-\rho} g_t}{1 - \rho} + (1 - n) \frac{C_t^{1-\rho} g_t^*}{1 - \rho} = n\bar{C}^{-\rho}(C_t - \bar{C}) + (1 - n)\bar{C}^{-\rho}(C_t^* - \bar{C}) + \]

\[ \frac{n}{2} \bar{C}^{-\rho -1} (C_t - \bar{C})^2 + \frac{1 - n}{2} \bar{C}^{-\rho -1} (C_t^* - \bar{C})^2 + n\bar{C}^{-\rho}(C_t - \bar{C})(g_t - 1) \]

\[ + (1 - n)\bar{C}^{-\rho}(C_t^* - \bar{C})(g_t^* - 1) + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3) \]  

(B.7)

where \( \mathcal{O}(\|\xi\|^3) \) represents all the terms that are of third order or higher in the deviations of the various variables from their steady-state values and in t.i.p. we include all the terms that are independent of monetary policy. Furthermore, expanding \( C_t, \ C_t^*, \ C_t^W \) with a second-order Taylor approximation we obtain

\[ C_t = \bar{C}(1 + \hat{C}_t + \frac{1}{2}\hat{C}_t^2) + o(\|\xi\|^3), \]

(B.9)

\[ C_t^* = \bar{C}(1 + \hat{C}_t^* + \frac{1}{2}\hat{C}_t^{*2}) + o(\|\xi\|^3), \]

(B.10)

\[ C_t^W = \bar{C}(1 + \hat{C}_t^W + \frac{1}{2}(\hat{C}_t^W)^2) + o(\|\xi\|^3), \]

(B.11)
where $\hat{C}_t = \ln(C_t/\bar{C})$, $\hat{C}_t^* = \ln(C_t^* / \bar{C})$ and $\hat{C}_t^W = \ln(C_t^W / \bar{C})$. Substituting (B.9), (B.10) and (B.11) into (B.8) we obtain that

$$n \frac{C_t^{1-\rho} g_t}{1-\rho} + (1-n) \frac{C_t^{*1-\rho} g_t^*}{1-\rho} = \hat{C}_t^1 + \frac{1}{2} (\hat{C}_t^W)^2 - n \frac{\partial}{2} \hat{C}_t^2$$

$$-(1-n) \frac{\partial}{2} (\hat{C}_t^*)^2 + n \hat{C}_t^W \hat{g}_t + (1-n) \hat{C}_t^* \hat{g}_t + \text{t.i.p.} + O(\|\xi\|^3). \quad (B.12)$$

Similarly, we take a second-order Taylor expansion of $\frac{y_t(h)^{1+\eta z_t^{-1}(1+\eta)}}{1+\eta}$ around a steady state where $y_t(h) = \bar{Y} = \bar{C}$ for each $h$, at each date $t$, and where $z_t = 1$ at each date $t$, obtaining

$$\frac{y_t(h)^{1+\eta a_t^{-1}(1+\eta)}}{1+\eta} = \bar{Y} (y_t(h) - \bar{Y}) + \frac{1}{2} \bar{Y}^\eta (y_t(h) - \bar{Y})^2$$

$$+\bar{Y} (1 + \eta)(y_t(h) - \bar{Y}) (z_t - 1) + \text{t.i.p.} + O(\|\xi\|^3). \quad (B.13)$$

In the same way, we can take an expansion of $\frac{y_t^*(f)^{1+\eta z_t^{-1}(1+\eta)}}{1+\eta}$ obtaining

$$\frac{y_t^*(f)^{1+\eta a_t^{-1}(1+\eta)}}{1+\eta} = \bar{Y} (y_t^*(f) - \bar{Y}) + \frac{1}{2} \bar{Y}^\eta (y_t^*(f) - \bar{Y})^2$$

$$+\bar{Y} (1 + \eta)(y_t^*(f) - \bar{Y}) (z_t^* - 1) + \text{t.i.p.} + O(\|\xi\|^3). \quad (B.14)$$

Combining (B.13) and (B.14), we obtain after some algebra that

$$\int_0^n \frac{y_t(h)^{1+\eta z_t^{-1}(1+\eta)}}{1+\eta} dh + \int_n^1 \frac{y_t^*(f)^{1+\eta z_t^{-1}(1+\eta)}}{1+\eta} df = \bar{Y}^{1+\eta} [\hat{C}_t^W + \frac{1}{2} (\hat{C}_t^W)^2 + n E_h [\hat{y}_t(h)] + (1-n) E_f [\hat{y}_t(f)]$$

$$+n(1-n) \theta(\hat{T}_t)^2 + \frac{n}{2} E_h [\hat{y}_t(h)]^2 + \frac{1-n}{2} E_f [\hat{y}_t(f)]^2 + n \frac{\eta}{2} E_h [\hat{y}_t(h)]^2$$

$$+(1-n) \frac{\eta}{2} E_f [\hat{y}_t(f)]^2 - n(1+\eta) \cdot \hat{z}_t E_h \hat{y}_t(h) - (1-n)(1+\eta) \cdot \hat{z}_t^* E_f \hat{y}_t(f)$$

$$+\text{t.i.p.} + O(\|\xi\|^3). \quad (B.15)$$

Note that

$$E_h [\hat{y}_t(h)]^2 = \text{var}_h \hat{y}_t(h) + [E_h \hat{y}_t(h)]^2, \quad (B.16)$$

$$E_h [\hat{y}_t(f)]^2 = \text{var}_h \hat{y}_t(f) + [E_h \hat{y}_t(f)]^2, \quad (B.17)$$
We can define the aggregators
\[ Y_{H,t} = \left[ \left( \frac{1}{n} \int_{0}^{n} y(h) \frac{\partial}{\partial h} dh \right) \frac{\partial}{\partial t} \right] = \left( \frac{P_{H,t}}{P_{t}} \right)^{-\theta} C_{t}^{W}, \]
\[ Y_{F,t} = \left[ \left( \frac{1}{1-n} \int_{n}^{1} y(f) \frac{\partial}{\partial f} df \right) \frac{\partial}{\partial t} \right] = \left( \frac{P_{F,t}}{P_{t}} \right)^{-\theta} C_{t}^{W}, \]
and take a second-order approximation of them obtaining
\[ \hat{Y}_{H,t} = E_{h} \hat{y}_{h}(h) + \frac{1}{2} \left( \frac{\sigma - 1}{\sigma} \right) \text{var}_{h} \hat{y}_{h}(h) + O(\|\xi\|^3), \quad \text{(B.18)} \]
\[ \hat{Y}_{F,t} = E_{f} \hat{y}_{f}(f) + \frac{1}{2} \left( \frac{\sigma - 1}{\sigma} \right) \text{var}_{f} \hat{y}_{f}(f) + O(\|\xi\|^3). \quad \text{(B.19)} \]

Finally substituting (B.16), (B.17), (B.18) and (B.19) into (B.15) we obtain,
\[ \int_{0}^{n} \frac{y_{h}(h)^{\eta}z_{t}^{\eta}z_{t}^{-\eta}}{1 + \eta} dh + \int_{n}^{1} \frac{y_{f}(f)^{\eta}z_{t}^{\eta}z_{t}^{-\eta}}{1 + \eta} df = \bar{C}^{1-\rho} \cdot [\hat{C}_{t}^{W} + \frac{1}{2}(\hat{C}_{t}^{W})^{2}] + n(1-n)\theta \bar{T}_{t}^{2} + \frac{1}{2} \eta \cdot [n \hat{Y}_{H,t} + (1-n)\hat{Y}_{F,t} + (1+n)\hat{Y}_{t} - n \hat{Y}_{H,t} + (1-n)\hat{Y}_{F,t}] \\
+ \frac{1}{2} (\sigma^{-1} + \eta) \cdot [n \text{var}_{h} \hat{y}_{h}(h) + (1-n)\text{var}_{f} \hat{y}_{f}(f)] + \text{t.i.p.} + O(\|\xi\|^3). \quad \text{(B.20)} \]

where we have used the fact that \( \text{var}_{h} \hat{y}_{h}(h) = \text{var}_{f} \hat{y}_{f}(h) \) and \( \text{var}_{f} \hat{y}_{f}(f) = \text{var}_{f} \hat{y}_{f}(f) \).

Combining (B.12), (B.20) and (B.6) and substituting the expressions for \( \hat{Y}_{H,t}, \hat{Y}_{F,t} \) after some algebra we get
\[ w_{t} = -\bar{C}^{1-\rho} \cdot \left\{ \frac{1}{2}(\rho + \eta)[\hat{C}_{t}^{W} - \bar{C}_{t}^{W}]^{2} + \frac{1}{2} n(1-n)\rho[\hat{C}_{t}^{R} - \bar{C}_{t}^{R}]^{2} \right\} + \frac{1}{2} n(1-n)(1+\eta\theta)\theta[\bar{T}_{t} - \bar{T}_{t}]^{2} + \frac{1}{2} (\sigma^{-1} + \eta) \cdot [n \text{var}_{h} \hat{y}_{h}(h) + (1-n)\text{var}_{f} \hat{y}_{f}(f)] + \text{t.i.p.} + O(\|\xi\|^3). \]

after having used the definitions of \( \hat{C}_{t}^{W}, \bar{C}_{t}^{R} \) and \( \bar{T}_{t} \). Using the definition of consumption gap, we obtain
\[ w_{t} = -\bar{C}^{1-\rho} \cdot \left\{ \frac{1}{2}(\rho + \eta)[c_{t}^{W}]^{2} + \frac{1}{2} n(1-n)\rho[c_{t}^{R}]^{2} + \frac{1}{2} n(1-n)(1+\eta\theta)\theta[\bar{T}_{t} - \bar{T}_{t}]^{2} \right\} + \frac{1}{2} (\sigma^{-1} + \eta) \cdot [n \text{var}_{h} \hat{y}_{h}(h) + (1-n)\text{var}_{f} \hat{y}_{f}(f)] + \text{t.i.p.} + O(\|\xi\|^3), \]

iv
Following Woodford (2003), we derive \( \text{var}_h \tilde{y}_t(h) \) and \( \text{var}_f \tilde{y}_t(f) \) to get
\[
\sum_{t=0}^{\infty} \beta^t \text{var}_h \{ \log y_t(h) \} = \frac{\alpha}{(1 - \alpha)(1 - \alpha \beta)} \sigma^2 \sum_{t=0}^{\infty} \beta^t (\pi_{H,t})^2 + \text{t.i.p.} + O(\|\xi\|^3),
\]
\[
\sum_{t=0}^{\infty} \beta^t \text{var}_f \{ \log y_t(f) \} = \frac{\alpha^*}{(1 - \alpha^*)(1 - \alpha^* \beta)} \sigma^2 \sum_{t=0}^{\infty} \beta^t (\pi_{F,t}^*)^2 + \text{t.i.p.} + O(\|\xi\|^3)
\]
We finally obtain
\[
W = -\frac{1}{2} U C C E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}
\]
where
\[
L_t = (\rho + \eta) \cdot [c_t^W]^2 + n(1 - n) \rho [c_t^R]^2 + n(1 - n)(1 + \eta \theta) \cdot [\hat{g}_t - \tilde{T}_t]^2 \\
+ n \frac{\sigma}{k} (\pi_{H,t})^2 + (1 - n) \rho \frac{\sigma}{k^*} (\pi_{F,t}^*)^2 + \text{t.i.p.} + O(\|\xi\|^3),
\]
which corresponds to equation (14) in the text.

**The case with a non-zero steady-state net foreign asset holdings.**

Here we analyze the case in which the steady-state net foreign asset holdings are different from zero. Recalling the definition of world consumption
\[
C_t^W = n C_t + (1 - n) C_t^*,
\]
we observe that in a first-order approximation can be written as
\[
\hat{C}_t^W = s \hat{C}_t + (1 - s) \hat{C}_t^* + O(\|\xi\|^3),
\]
where \( s \) is the share of the domestic consumption in world consumption, i.e \( s \equiv n \bar{C} / \bar{C}^W \)
It follows that
\[
\hat{C}_t = \hat{C}_t^W + (1 - s) \hat{C}_t^R + O(\|\xi\|^3),
\]
\[
C_t^* = C_t^W - s \hat{C}_t^R + O(\|\xi\|^3),
\]
where \( \hat{C}_t^R \equiv \hat{C}_t - \hat{C}_t^* \).

The first change occurs in the flexible-price complete-market allocation in its log-linear approximation
\[
\hat{C}_t^R = \frac{1}{\rho} \hat{g}_t^R,
\]
\[
\tilde{C}_t^W = \frac{1}{\rho + \eta} \left[ \hat{g}_t^W + (s - n) \hat{g}_t^R \right] + \frac{\eta}{\rho + \eta} \tilde{z}_t^W,
\]
\[
\tilde{T}_t = \frac{\eta}{1 + \theta \eta} \tilde{z}_t^R.
\]
In table 2, the only equations that will be affected by the asymmetries in the steady-state foreign asset holdings are the IS\(^W\), CA\(^I\), AS\(_H\) and AS\(_H\). In particular IS\(^W\) becomes
\[
\rho E_t(c_{t+1}^W - c_t^W) = s_t + (1 - s)\hat{r}_t - \hat{R}_t^W - E_t[n\pi_{H,t+1} + (1 - n)\pi_{F,t+1}^* + (s - n)\Delta S_{t+1}],
\]
where now
\[
\hat{R}_t^W = \rho E_t(\Delta \hat{w}_t^W) - E_t(\hat{g}_{t+1}^W + (s - n)\hat{g}_{t+1}^R - \hat{g}_t^W - (s - n)\hat{g}_{t}^R).
\]
The current account equation CA\(^I\) becomes
\[
(1 + \bar{a}_F\delta)\beta w_t = w_{t-1} + \bar{a}_F(\beta \hat{c}_t^* - \pi_t^*) + (\theta - 1)(1 - n)\hat{\theta}_t + \hat{C}_t^W - \frac{s_t}{n}\hat{C}_t,
\]
where \(s_t \equiv n(1 + (1 - \beta)\bar{a}_F)\) and \(w_t \equiv A_{F,t}/(P_t^*Y) - \bar{a}_F\). The two aggregate supply equations become
\[
\pi_{H,t} = k[(\rho + \eta)c_t^W + \rho(1 - s)c_t^R + (1 - n)(1 + \eta\theta)(\hat{T}_t - \bar{T}_t)] + \beta E_t\pi_{H,t+1},
\]
\[
\pi_{F,t}^* = k^*[(\rho + \eta)c_t^W - \rho sc_t^R - n(1 + \eta\theta)(\hat{T}_t - \bar{T}_t)] + \beta E_t\pi_{F,t+1}^*.
\]
Here we show how to derive equation (20) in the text, when \(C \neq \hat{C}^*\). We assume that in the steady state the marginal utilities of consumption are equated across countries, i.e.
\[
C^{-\rho}\hat{g} = C^*^{-\rho}\hat{g}^*.
\]
We take a second-order expansion of the term
\[
\frac{nC_{t+1}^{1-\rho}g_t}{1 - \rho} + (1 - n)\frac{C_t^{1-\rho}g_t^*}{1 - \rho}
\]
onobtaining after some steps
\[
\frac{nC_{t+1}^{1-\rho}g_t}{1 - \rho} + (1 - n)\frac{C_t^{1-\rho}g_t^*}{1 - \rho} = C^{-\rho}\hat{g}C^W[\hat{C}_t^W + \frac{1}{2}(\hat{g}_t^W)^2 - \frac{\rho}{2}(\hat{g}_t^W)^2]
\]
\[
-s(1 - s)\frac{\rho}{2}(\hat{C}_t^R - \rho^{-1}\hat{R}_t^R)^2 + \rho C_t^W(s\hat{g}_t + (1 - s)\hat{g}_t^*)] + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3), \tag{B.21}
\]
The second-order expansion of the disutilities of working will not be affected by the different steady-state level of consumption across countries. We continue to assume that the monopolistic distortions are completely offset by appropriate taxation subsidies in both countries.
Combining (B.20) and (B.21) and repeating the steps of the previous section, we can get
\[
W = -\frac{1}{2}U CC^W E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}
\]
where

\[ L_t = (\rho + \eta) \cdot [c_t^W]^2 + s(1 - s)\rho [c_t^R]^2 + n(1 - n)(1 + \eta \theta) \cdot [\hat{T}_t - \tilde{T}_t]^2 \]
\[ + n\frac{\sigma}{k} (\pi_{H,t})^2 + (1 - n) \frac{\sigma}{k^*} (\pi_{F,t}^*)^2 + \text{t.i.p.} + o(||\xi||^3), \]

which corresponds to equation (20) in the text.
Table 3:
Welfare Costs of Incomplete Markets under Producer-Price Stability
(% of a permanent shift in steady state consumption)

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Productivity Shocks Only

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Table 4:

Welfare Gains by Using the Optimal Monetary Policy
All shocks
(\% of a permanent shift in steady state consumption)

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Table 5:
Welfare Costs of Incomplete Markets under Producer-Price Stability
with Non-Zero Initial Holdings of Foreign Assets
(% of a permanent shift in steady state consumption)

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Table 6:
Gains by Using the Optimal Cooperative Monetary Policy
with Non-Zero Initial Holdings of Foreign Assets
(% of a permanent shift in steady state consumption)

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<tr>
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Figure 1: Comparisons between price-stability and optimal policy: debt denominated in foreign currency
Figure 2: Comparisons between price-stability and optimal policy: debt denominated in foreign currency
Figure 3: Comparisons between price-stability and optimal policy: debt denominated in domestic currency
Figure 4: Comparisons between price-stability and optimal policy: debt denominated in domestic currency
Figure 5: Comparisons between price-stability and optimal policy: debt denominated in domestic currency
Figure 6: Comparisons between price-stability and optimal policy: debt denominated in domestic currency