Comment on “Fiscal Externalities and Optimal Taxation in an Economic Community,” by Marianne Baxter and Robert G. King

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Optimal taxation problems in open-economy models are not trivial extensions of similar analyses in closed-economy models. In this discussion, I emphasize two important caveats of open-economy optimal policy problem. First, the intertemporal budget constraint of the government is not a necessary restriction that a Ramsey government should consider in its maximization problem. This point revisit in the current framework a previous argument discussed in Woodford (1996). Second, the specification of the strategies followed by each government is critical for the outcome of the non-cooperative allocation. Finally, I argue that in the analysis of Baxter and King (2005), it would be interesting to know more about how the labor wedge is set across the different allocations and models analyzed.

Intertemporal budget constraint of the government in open economy models

In the so-called Ramsey’s approach to optimal taxation, the government chooses taxes in order to maximize the utility of the households under the sequence of resource constraints of the economy and the constraints implied by the optimizing behavior of households. Government is then ‘benevolent’. It would seem natural to assume that a relevant constraint for this optimal policy problem is the intertemporal budget constraint of the government. But, in the Ramsey’s approach, this can be justified only if this constraint belongs either to the resource constraints of the economy or is an implication of the optimizing behavior of households.

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In this section I show that the intertemporal budget constraint of the government is a relevant constraint for the optimization problem of a ‘Ramsey’ government in a closed-economy model but not in an open-economy model.

Consider the closed-economy model presented in BK where households maximize

\[ U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \]  

in which \( \beta \) is the discount factor with \( 0 < \beta < 1 \) and \( u(\cdot) \) is the utility function of consumption, \( c \), and labor, \( n \), with standard properties. Households are subject to a flow budget constraint of the form

\[ \frac{b_{t+1}^p}{1 + r_t} = b_t^p + (1 - \tau_t^n)w_t n_t - (1 + \tau_t^c)c_t. \]  

At time \( t \), households’ revenues are given by labor income, where \( w_t \) are wages taxed at the rate \( \tau_t^n \), and by the value of the financial assets carried over from the previous period, \( b_t^p \); consumption is also taxed at the rate \( \tau_t^c \). Households can borrow or lend at time \( t \) using an asset, \( b_{t+1}^p \), which is issued at discount and gives a return \( r_t \). An initial condition on the assets at time 0 is given, \( b_0 = 0 \). The constraint (2) is not enough to impose a well-defined maximization problem, for consumption can be infinite. A natural borrowing limit conditions of the form

\[ b_t^p \geq -\sum_{T=t}^{\infty} R_{t,T}(1 - \tau_T^n)w_T n_T > -\infty \]  

is added, stating that households’ borrowing in a certain period cannot exceed the present discounted value, net of taxes, of wage revenues discounted by the appropriate factor defined as

\[ R_{t,T} \equiv \prod_{s=t}^{T-1} \left( \frac{1}{1 + r_s} \right) \]

for each \( T > t \) while \( R_{t,t} \equiv 1 \). Household’s optimization problem is to maximize (1) under the sequences of flow budget constraint (2) and the borrowing-limit constraints (3), given the initial condition \( b_0 \). This maximization problem has two other equivalent formulations. In the first, the utility is maximized under the sequences of flow budget constraint (2) and the intertemporal budget constraint of the households

\[ \sum_{t=0}^{\infty} R_{0,t}(1 + \tau_t^c)c_t \leq \sum_{t=0}^{\infty} R_{0,t}(1 - \tau_t^n)w_t n_t. \]
In the second, the utility is maximized under the sequences of flow budget constraints (2) and the transversality condition

$$\lim_{T \to \infty} R_{0,T} b_T^p \geq 0.$$ 

In this economy, the government is subject to a flow budget constraint of the form

$$\frac{b_{t+1}^g}{1 + r_t} = b_t^g + \tau_t^n w_t n_t + \tau_t^c c_t - g_t \quad (4)$$

in which $b_t^g$ denotes government assets at time $t$ carried over from the previous period; $g_t$ represents exogenous government purchases of the only good produced in this economy. Government can borrow and lend freely from the private sector. Goods are produced in the economy with a technology of the form $y_t = a_t n_t$ where $a_t$ is an exogenous productivity shock. Equilibrium in the goods and assets markets requires that

$$y_t = c_t + g_t, \quad (5)$$

$$b_t^p + b_t^g = 0, \quad (6)$$

respectively.

Firm’s optimization problem implies that wages are equalized to productivity

$$w_t = a_t$$

Optimality conditions on the side of the consumers imply that: 1) the marginal rate of substitution between consumption and labor is equated to the labor wedge

$$\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = \frac{(1 - \tau_t^n)}{(1 + \tau_t^c)} w_t; \quad (7)$$

2) the marginal utilities of consumption between subsequent periods are related through the following Euler equation

$$\frac{u_c(c_t, n_t)}{(1 + \tau_t^c)} = \beta(1 + r_t) \frac{u_c(c_{t+1}, n_{t+1})}{(1 + \tau_{t+1}^c)}; \quad (8)$$

3) the intertemporal budget constraint of the consumer is satisfied with equality

$$\sum_{t=0}^{\infty} R_{0,t}(1 + \tau_t^c) c_t = \sum_{t=0}^{\infty} R_{0,t}(1 - \tau_t^n) w_t n_t \quad (9)$$
or alternatively the transversality condition is satisfied with equality

\[ \lim_{T \to \infty} R_{0,T} b_p^T = 0. \quad (10) \]

As a consequence of this optimizing behavior, (9) together with the resource constraint of the economy

\[ y_t = a_t n_t = w_t n_t = g_t + c_t \]

implies the intertemporal budget constraint of the government

\[ \sum_{t=0}^{\infty} R_{0,t} [\tau^c_t c_t + \tau^w_t w_t n_t] = \sum_{t=0}^{\infty} R_{0,t} g_t \]

and viceversa (12) and (11) imply (9).

Although the government is only subject to a flow budget constraint of the form (4), the intertemporal budget constraint of the government is also an equilibrium condition as a consequence of the optimizing behavior of the consumers in this economy. The intertemporal budget constraint of the government mirrors through (11) that of the households.

In an alternative interpretation, the flow budget constraint (4) is not enough to imply that (12) holds. It would be the case were a transversality condition of the form

\[ \lim_{T \to -\infty} R_{0,T} b_p^T = 0 \]

holding. And indeed this is the case since the equilibrium in the asset markets (6) together with (10) implies (13).

In a multi-country open-economy model, this implication does not hold. Consider, for simplicity, a two-country (home and foreign) version of the above model. In this case, equilibrium in the goods and asset markets requires that

\[ y_t + y_t^* = c_t + c_t^* + g_t + g_t^* \]

\[ b_p^T + b_p^T + b_p^T + b_p^T = 0 \]

where starred variables denote the respective variables for the foreign country. Conditions (7), (8), (9) and (10) hold for the home and foreign households. However, it is not the case that (12) holds for each country. Indeed the only implication of condition (10)
together with the respective condition for the foreign households and the equilibrium condition (15) is that

$$\lim_{T \to \infty} R_{0,T}(b_t + b_t^g) = 0$$

and then that an intertemporal budget constraint of the government holds at an aggregate level, i.e.

$$\sum_{t=0}^{\infty} R_{0,t} [\tau_t c_t + \tau_t^* c_t^* + \tau_t^n w_t n_t + \tau_t^* n_w t n_t^*] = \sum_{t=0}^{\infty} R_{0,t} [g_t + g_t^*].$$

As a consequence of the possible violation of the intertemporal budget constraint of the government at the country level, it is not necessarily the case that the intertemporal resource constraints

$$\sum_{t=0}^{\infty} R_{0,t} (y_t - g_t - c_t) = 0 \quad \sum_{t=0}^{\infty} R_{0,t} (y_t^* - g_t^* - c_t^*) = 0$$

hold for each country, as it is instead assumed in BK. A benevolent Ramsey central planner that maximizes the aggregate utility of the households belonging to this union might not necessarily find optimal to obey to (16) in its optimal plan. Violation of (16) might even be possible in the non-cooperative allocation.

It would be interesting to investigate whether there are cases in which a central planner would prefer that (16) holds for both countries while in the non-cooperative allocation (16) would be instead violated. Perhaps it is even possible to argue for cases in which the equilibrium allocation for the endogenous variables that results from a strategic game can be non stationary, even though exogenous disturbances are assumed stationary.

**Strategy spaces in open-economy models**

In the characterization of the optimal policy problem of a closed-economy model it does not really matter whether the instrument of policy is specified or not, unless this specification represents a constraint on the set of possible equilibrium allocations. Instead, in an open-economy problem, this can be an important issue and it is moreover critical when non-cooperative allocations are analyzed. Indeed, non-cooperative allocations depend on the concept of strategic equilibrium assumed and in particular on the strategies specified for each of the two governments. As an example, in a standard duopoly problem the equilibrium outcome is different whether prices or quantities are
assumed as strategies. In this paper, it is assumed that the strategy of a generic government \( j \) is specified in terms of the variables \( \xi_{jt}, \xi_{jt}^c \). In particular \( \xi_{jt}^n, \xi_{jt}^c \) are defined as

\[
\xi_{jt}^n = \Lambda_j (1 - \tau_{jt}^n) \\
\xi_{jt}^c = \Lambda_j (1 + \tau_{jt}^c)
\]

where

\[
\Lambda_j = \Lambda_j (\{\tau_{jt}^n\}, \{\tau_{jt}^c\}, \{\delta_t\}, \{w_t\}).
\]

Moreover \( \delta_t \) depends also on the tax rates \( \tau_{it}^n, \tau_{it}^c \) for each \( i \neq j \). A game in which each government chooses optimally \( \xi_{jt}^n, \xi_{jt}^c \) under the constraints of the economy taking as given \( \xi_{it}^n, \xi_{it}^c \) for each other government \( i \), is likely not to correspond to a game in which the strategy space is specified in terms of \( \tau_{jt}^n \) and \( \tau_{jt}^c \). Indeed in the latter case, government in country \( i \) can internalize the effect of its action on the variable \( \Lambda_j \) of country \( j \), while this is precluded in the former case. Whether there is equivalence between the outcomes of these two games should be proved. The game in which the strategies are specified in terms of the tax rates \( \tau_{jt}^n, \tau_{jt}^c \) seems the one relevant for policy analysis. Indeed one might wonder how it is possible to assume that each government decides on the variables \( \xi_{jt}^n, \xi_{jt}^c \) which are then non-linear functions of the tax rates, even of those of the other countries. It is important to note that this device is helpful to get an analytical solution, which otherwise I doubt it will be possible.

**Other comments**

An interesting result, robust to the closed and open-economy versions of the model, is the fact that the labor wedge is required to be constant across time. The labor wedge, \( k \), is given by the following expression

\[
k = \frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} \frac{1}{w_t} = \frac{1 - \tau_{it}^n}{1 + \tau_{it}^c}
\]

and measures the distortions existing in this economy. The fact that this wedge is constant across the different specifications requires further investigation. In particular, it seems that an important aspect to know is the level at which this wedge is set across the different models and allocation. First, it would be interesting to know whether having a small-open economy model implies a different \( k \) with respect to the closed-economy case. Moreover, the size of the labor wedge would matter in the comparison between
the cooperative and non-cooperative allocation for the evaluation of the magnitude of the externalities and gains from cooperation. Given the constant-labor-wedge result, the paper focuses on the fact that sometimes the single tax rates $\tau^n_i$ and $\tau^c_i$ can be time varying. However, I suspect that the time-varying properties of the tax rates is likely to be of second-order importance with respect to the wedge differences, if any. And indeed, fiscal policy is usually thought to have level (or structural) effect on the equilibrium allocation. The paper is silent on whether there is such a role of fiscal policy that comes out from this model and more research on this issue is needed.

References
