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THE VALUATION OF ITALIAN FLOATING RATE TREASURIES

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SUMMARY

The paper describes a method for valuing floating rate notes. A short introduction surveying the literature in this field and outlining the organization of the work is followed by a description of the features of floating rate Italian Treasury credit certificates (CCTs) and a breakdown of their prices. The following section puts forward and analyzes a simple generalization of the formula for valuing elementary floating rate coupons associated with the one-factor Cox, Ingersoll and Ross model. Specifically, the paper highlights the special properties of such mini-securities, which, depending on their maturity, react positively or negatively to changes in the state variable of the model (the instantaneous interest rate). Some extensions of the valuation formula then make it possible to take account of the specific characteristics of the reference parameter provided for in the rules governing issues of CCTs. Finally, the daily prices of these securities are analyzed over a period of around ten years. The empirical test makes it possible to estimate a variable that measures the extra risk the market associates with CCTs. The changes in this variable over time suggest a number of considerations concerning operators' perception of this risk.

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1. INTRODUCTION

The literature on the valuation of floating rate notes is not very large and the first theoretical contribution was made at the start of the eighties. More recently, the valuation of such securities has only been studied in connection with interest rate swaps, with the focus on the variable leg.

Cox Ingersoll and Ross [CIR; 1980], were the first to apply the instruments of modern finance to floating rate instruments in continuous time. In the first place they examined the immunization properties of such securities on the hypothesis of continuous and discrete payments; then, assuming a dynamic for the instantaneous interest rate of the type $dr = sr^{3/2}dz$, they obtained closed form solutions to the problem of valuing floating rate consols with various contractual conditions (minimum or maximum coupon, early redemption option, etc.). Finally, they extended the analysis to the case in which there is a credit risk of the issuer.

Ramaswamy and Sundaresan [1986] started by describing the features of the floating rate notes in circulation in the US market and, by examining their prices, revealed the substantial discount compared with their face value; assuming a dynamic of the type $dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dz$ (square root model) for the instantaneous interest rate, they then considered various contractual conditions in numerically valuing a five-year security with a continuous coupon equal to an exponential mean of the instantaneous rates observed previously. Finally, since the model was unable to explain the large price discounts,¹ they introduced a second state variable consisting of the instantaneous premium for the risk of the issuer becoming insolvent and concluded that the discounts observed could only be explained on the hypothesis of much smaller spreads than those required by the market.²

Barone and Cesari [1986], after using a two-stage procedure to estimate the same one-factor model of the square root type, used forward approximation³ to value floating rate Italian Treasury credit certificates (CCTs). The model was unable to explain the large discounts present in the prices of these securities; the extra riskiness, extraneous to the model, was quantified by considering the price elasticity of the basis risk.⁴

Castellani, De Felice and Moriconi [1989], used the same square root model to derive a closed-form solution to the problem of valuing stochastic zero coupon bonds, i.e. of securities with a single floating rate elementary coupon. They then broke down the risk on such securities into two components (the discount risk and the coupon risk).

Sundaresan [1991a] put forward, in the same theoretical context, a closed-form formula for the valuation of floating rate elementary coupons. He then extended the result to the case of a two-factor model in which the second state variable is represented by the instantaneous premium against the risk of the issuer becoming insolvent.

This paper starts with a description of the contractual features of CCTs and then points out the difformities compared with the theoretical prescriptions (§2.1). The theoretical value of CCTs is then represented as the sum of both deterministic and uncertain future payments, discounted using the appropriate factors (§2.2).

The next section describes the theoretical environment for the valuation of CCTs, with special reference to the floating rate coupons. After introducing the forward risk-adjusted process, defined by the Separation Theorem (§3.1), a simple generalization of the formula for valuing an elementary floating rate coupon is presented (§3.2). Specifically, the formula permits the valuation of floating rate coupons when the base yield is forward and quoted for an interval different from the reference period (e.g. an annual yield which has to be transformed, under compound interest, into the semi-annual equivalent).

¹. According to Ramaswamy and Sundaresan [1986; p. 266] "... it seems unlikely that lags or contractual features will serve to explain the reported prices"

². For a different analysis of the effects of credit risk on the price and duration of floating rate notes, see Yawitz, Kaufold, Macirowsky and Smirlock [1987]. In particular, the following result should be noted: "... the value of a floater may increase when other interest rates rise"

³. Forward approximation consists in determining the current value of the variable rate coupon by discounting the appropriate forward interest rate implicit in the current term structure (cfr. §3.4).

⁴. Basis risk is taken to be the risk deriving from changes in the instantaneous interest rate.

A sensitivity analysis is then carried out to test the effects on the value of the floating rate coupons of changes in the state variable (the instantaneous interest rate), the parameters of the model and the contractual features of these elementary securities (§3.3). A test is then made of the accuracy of the forward approximation used in other studies (§3.4).

The third section first extends the formula for valuing elementary coupons to the case of CCTs linked to the gross yield on Italian T-bills. This is necessary when the pre-tax prices of T-bills are modeled (§4.1). Besides, in order to obtain a valuation formula in the case of a link with average yields, portfolios consisting of several coupons with the same payment date are considered (§4.2).

To permit the model to express below par theoretical values of CCTs, even with large spreads with respect to the reference parameter, an additional parameter has been added that captures the extra riskiness perceived by operators, especially as regards the future representativeness of the base yields (§4.3).

The fifth section consists of an extensive empirical test. After describing the sample (§5.1 and the estimation methodology (§5.2), we show the coefficients and residuals of the regressions, respectively in sections (§5.3) and (§5.4). Finally, a test is made of the correspondence between the T-bill yields implicit in the prices of CCTs and those observed at auction (§5.5).

The paper ends with a number of conclusions.

2. ITALIAN TREASURY CREDIT CERTIFICATES (CCTs)

2.1 Features

The features of CCTs, which were first issued in July 1977,⁵ have changed over the years as a result of changes in both issue rules (linkage mechanism, coupon interval, reference rate and implicit options) and tax law (Figure 1). The methods used for placing these securities have also changed.

Table a2 in the Appendix provides a summary view of the nine different categories of CCT offered to investors and identified hereinafter by the symbols $\Psi_1, \Psi_2, \dots, \Psi_9$.⁶

For each type of linkage the Table shows the interval of the CCT coupons, the period fixed for their determination, the maturity of the T-bills to which they refer, the method of determining the price and base yield of the T-bills, the methods used to determine the reference parameter and the calculation of the gross and net coupon.

An example of the process of determining the coupons is given in Table a3 and Table a4. Table a3 shows the dates involved in valuing a coupon:

- s_j : the date on which j -th coupon is paid;
- u_{jk} : the date on which the result of the k -th auction of the d included in the reference period;
- v_{jk} : the date on which settlement is made for the T-bills bought;
- T_{jk} : the date on which the T-bills are redeemed.

In turn, Table a4 shows the various steps needed to determine the value of the coupons. The four T-bill auctions held in May and June 1990 are considered, with examples of the calculations required to determine the coupons of six different types of linkage ($\Psi_3, \Psi_4, \Psi_5, \Psi_6, \Psi_7$ e Ψ_9).

The features of the CCTs currently issued (tipologia Ψ_9) differisce per diversi aspetti dalle prescrizioni teoriche. differ in several respects from the theoretical prescriptions. The main differences are:

- a) the base yield, represented by the 12-month T-bill auction rate, is directly controllable by the issuer. In the purely hypothetical case of a gradual reduction in the quantities offered, the interest rate on 12-month T-bills could lose its significance.⁷ In the extreme case of the Treasury

⁵ For an analysis of the economic policy motives that led to their introduction, see Masera [1977].

⁶ The tables are all in the Appendix. The main symbols used in this paper are shown in Table a1.

⁷ The Treasury's supply policy for T-bills in the period 1977-1991 is shown in Table a5.

Figure 1 The features of Italian Treasury Certificates (1977 - 1991)

suspending issues of 12-month T-bills, CCTs would be comparable to fixed rate securities since the coupons would be determined with reference to the last yield observed at auction.⁸

This risk, which would not exist if the base yield were an interbank rate, almost certainly entails a substantial additional cost for the Treasury. The absence of a perfect correlation be-

⁸ This point emerged in a conversation with Professor Luigi Spaventa. The issue rules explicitly provide that "In the event that in both months recourse is not made to the issue of 12-month T-bills, the rate will be equal to the last annual rate available".

tween the rate used for the linkage and the appropriate short-term rate conflicts with the immunization objective pursued through these securities.

- b) the maturity of the base yield (12 months) does not coincide with the coupon interval (6 months). This leads to positive or negative price corrections depending on whether the term structure of interest rates has an upward or a downward slope;
- c) the reference parameter is the arithmetic mean of the base yields observed in the four auctions of the two months that precede by one month that in which entitlement to the coupon begins (Table a3). The introduction of the “arithmetic mean” operator distorts the valuation process by requiring price corrections that can be negative or positive depending on the shape of the term structure. These corrections would not be necessary if the reference parameter were observed immediately before the date on which entitlement to the coupon begins
- d) The introduction of the “arithmetic mean” operator distorts the valuation process by requiring price corrections that can be negative or positive depending on the shape of the term structure. These corrections would not be necessary if the reference parameter were observed immediately before the date on which entitlement to the coupon begins⁹ and the security is issued below par. The spread also serves to compensate holders for the risk that the reference parameter will lose its significance over time, i.e. become less closely correlated with the optimal parameter. The spread remains unchanged for the whole life of the loan; it is a predetermined percentage that is added to the reference rate and which does not change as the latter varies. Since the spread does not diminish as the maturity of the security approaches, even though the risks that justify its existence almost certainly become smaller, prices normally tend to be above par close to the maturity of the security.

With the benefit of hindsight, it is clear that if the imperfections of points b) and c) had been eliminated by linking CCTs with semi-annual (annual) coupons to the yield of 6-month (12-month) T-bills observed at the auction immediately preceding the start of coupon entitlement, the Treasury would have simplified the linkage mechanisms, thereby reducing operators’ information costs while incurring an extra interest cost that, for a given first coupon, spread, tax treatment and set of contractual features (minimum coupon), would not have been greater than that actually incurred (Table a6).

The present mechanism benefits the Treasury when interest rates are rising, but imposes a penalty when they are falling. In 1990, for instance, the rise in interest rates during 1989 would have increased the interest payments on CCTs by 901 billion lire if the simplified linkage mechanism had been in force, while in 1987, following a long period of falling rates, they would have been 1,119 billion less than they actually were.

2.2 Theoretical value

The theoretical value of CCTs, $P(r, t; \mathbf{s}, \mathbf{a})$ is given by the sum of the future net payments $\mathbf{a} = \{a_j\}$, receivable at the dates $\mathbf{s} = \{s_j\}$, discounted by the appropriate factors:

$$P(r, t; \mathbf{s}, \mathbf{a}) = \sum_{j=1}^m a_j P(r, t; s_j) \quad (1)$$

where $P(r, t; s_j)$ is the price at time t of a pure discount bond maturing at time s_j , and r is the instantaneous interest rate.

In the case of a security with a face value of one, the m net payments $\mathbf{a} = \{a_j\}$ are defined as follows:

$$a_j = \begin{cases} \bar{a}_j & j = 1, m_0 \\ \rho[(\bar{\alpha}_j + spread)(1 - \theta), \quad 5E - 6] & j = 1, m_0 + 1, \dots, m - 1 \\ \rho[1 - \theta(1 - P_0) + (\bar{\alpha}_m + spread)(1 - \theta), \quad 5E - 6] & j = m \end{cases} \quad (2)$$

where:

⁹ It would be more correct to speak of a mark-up, but the term spread has become established in the market.

- $\bar{a}_j$ are the known coupons (one or at the most two);
- $\bar{a}_j$ is the certainty equivalent at maturity of the (gross) reference parameter fixed by the h -th type of linkage (Ψ_h);
- θ is the tax rate on interest;
- $\rho(x,y)$ is an operator that rounds x to the nearest multiple of y ;
- 5E-6 is the notation used to indicate 0,000005;
- P_0 is the base price of the security at issue.

Equation (1) can be rewritten as follows:

$$\begin{aligned}
P(r, t; \mathbf{s}, \mathbf{a}) = & \sum_{j=1}^{m_0} \bar{a}_j P(r, t; s_j) + & (\text{coupons that are known}) \\
& + \sum_{j=m_0+1}^m \rho[\bar{a}_j(1-\theta), \quad 5E-6] P(r, t; s_j) + & (\text{coupons that are not known})^{10} \\
& + \sum_{j=m_0+1}^m \rho[\text{spread}(1-\theta), \quad 5E-6] P(r, t; s_j) + & (\text{spread}) \\
& + \rho[1-\theta(1-P_0), \quad 5E-6] P(r, t; s_m) & (\text{face value})
\end{aligned} \tag{3}$$

The theoretical value of a CCT is thus equal to the current value of four, deterministic or uncertain, components: the coupons that are known, the coupons that are not yet known, the spread and the face value.

3. FLOATING RATE COUPONS

3.1 The forward risk-adjusted process

To value CCTs recourse has been made to the Cox, Ingersoll and Ross one-factor model, which assumes the following dynamic for the instantaneous interest rate:

$$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}dz \tag{4}$$

where κ is the speed of adjustment of the interest rate r towards its long-run average μ , $\sigma\sqrt{r}$ is the volatility of the changes in r and dz is a standardized Wiener process.

Imposing the equilibrium condition known as the Local Expectations Hypothesis

$$E\left(\frac{dH}{H}\right) = r + \frac{\lambda r H_r}{H} dt \tag{5}$$

where $H = H(r, t)$ is the price of any financial asset that depends exclusively on the current term structure, $-\lambda$ is the market price of the risk and rH_r/H represents the elasticity of the price of the security with respect to r , it can be shown that H follows (in the interval between two successive coupons) the following stochastic differential equation:

$$\frac{1}{2}\sigma^2 r H_{rr} + [\kappa\mu - (\kappa + \lambda)r]H_r + H_t - rH = 0 \tag{6}$$

Equation (6) represents Kolmogorov's (inhomogeneous) backward equation not of the actual process (4) but of the risk-neutral process

$$d\bar{r} = [\kappa\mu - (\kappa + \lambda)\bar{r}]dt + \sigma\sqrt{\bar{r}}dz \tag{7}$$

In other words it is $\bar{r}$ that determines prices, not r . Moreover, it is evident from equation (7) that the three original variables κ , μ and λ are "substituted" by the two variables $\kappa\mu$ and $\kappa + \lambda$.

Equation (6) can be solved once the appropriate boundary conditions for $H(r, t; T)$, have been imposed, in accordance with the features of each financial asset.

¹⁰ With no spread.

For the spot price $P(r, t; T)$ of a pure discount bond with a residual maturity $\tau = T - t$, the solution of equation (6), subject to the boundary condition $P(r, T; T) = 1$ is

$$P(r, t; T) = A(t, T)e^{-B(t, T)r} \quad (8)$$

where

$$A(t, T) = \left[\frac{\phi_1 e^{\phi_2 \tau}}{\phi_2 (e^{\phi_1 \tau} - 1) + \phi_1} \right]^{\phi_3} \quad (9)$$

and

$$B(t, T) = \left[\frac{e^{\phi_1 \tau} - 1}{\phi_2 (e^{\phi_1 \tau} - 1) + \phi_1} \right] \quad (10)$$

$$\phi_1 = \sqrt{(\kappa + \lambda)^2 + 2\sigma^2} \quad (11)$$

$$\phi_2 = \frac{\kappa + \lambda + \phi_1}{2} \quad (12)$$

$$\phi_3 = \frac{2\kappa\mu}{\sigma^2}. \quad (13)$$

The basis risk relative to a pure discount bond, i.e. the proportional change in the price caused by an infinitesimal change in the instantaneous interest rate (with the sign changed), is given by (8) as:

$$-\frac{\frac{\partial P(r, t; T)}{\partial r}}{P(r, t; T)} = B(t, T) \quad (14)$$

Denoting the basis risk of a zero-coupon security by δ ,

$$\delta = -\frac{\frac{\partial P(r, t; \mathbf{a}, \mathbf{s})}{\partial r}}{P(r, t; \mathbf{a}, \mathbf{s})} \quad (15)$$

the stochastic duration of the security (D) is equal to the maturity of the pure discount bond with the same δ :

$$D = \frac{1}{\phi_1} \ln \left(1 - \frac{\delta}{1 + \delta} \right) \quad (16)$$

Equation (8) also enables the whole term structure of interest rates to be obtained as:

$$R(r, t; T) = -\frac{\ln[P(r, t; T)]}{T - t} \quad (17)$$

In particular:

$$R_\infty = \lim_{T \rightarrow \infty} R(r, t; T) = (\phi_1 - \phi_2)\phi_3 \quad (18)$$

The term structure can have a uniform upward slope, a uniform downward slope or a hump, depending on which of the following conditions is satisfied:

$$r < R_\infty \quad (19)$$

$$r > \frac{\kappa\mu}{\kappa + \lambda} \quad (20)$$

$$R_\infty \leq r \leq \frac{\kappa\mu}{\kappa + \lambda} \quad (21)$$

As regards the valuation of financial assets with a risky payoff, Jamshidian (1987) has shown that their current value is equal to the discounted value of the expected payoff provided the discounting is based on the price of a pure discount bond with the same maturity and the expected value is calculated with reference to the forward risk-adjusted process, characterized by a time varying drift:

$$d\bar{r} = \left\{ \kappa\mu - \left[(\kappa + \lambda) + \sigma^2 B(t, T) \right] \bar{r} \right\} dt + \sigma \sqrt{\bar{r}} dz \quad (22)$$

The Separation Theorem,¹¹ therefore holds, and the price of a financial asset can therefore be decomposed as follows:

$$H(r, t; T) = P(r, t; T) \hat{E}_{r,t} [H(\bar{r}, T; T)] \quad (23)$$

where the symbol $\hat{E}_{r,t}$ denotes the expectation conditioned on $\bar{r}(t) = r$ and $\hat{E}_{r,t} H(\bar{r}, T; T)$ represents the certainty equivalent of the risky payoff relative to the financial asset in question

3.2 Elementary coupons

Taking $a(r, t, u, s, \Psi)$ to denote the value at time t of a floating rate coupon that at time s pays an amount to be determined at time u on the basis of linkage type Ψ , we consider the case of an elementary linkage $\Psi_e(v, T, \xi, \omega)$ whereby the maturity value of the coupon at time s is equal to the interest rate $R(r, u, v, T, \xi, \omega)$ determined at time u ($u \leq v < T$ e $u \leq s$), relative to the period (v, T) and related to a time base different from the reference period by means of the coefficients ξ and ω :

$$a(r, s; u, s, \Psi_e) = R(r, u; v, T, \xi, \omega) = \left\{ \left[\frac{P(r, u; v)}{P(r, u; T)} \right]^\xi - 1 \right\} \omega \quad (24)$$

The value of the coupon at the time (u) of its determination is:

$$\begin{aligned} a(r, u; u, s, \Psi_e) &= P(r, u; s) R(r, u; v, T, \xi, \omega) = \\ &= P(r, u; s) \left\{ \left[\frac{P(r, u; v)}{P(r, u; T)} \right]^\xi - 1 \right\} \omega = \\ &= P(r, u; s) \left\{ \left[\frac{P(r, u; v)}{P(r, u; T)} \right]^\xi - P(r, u; s) \right\} \omega \end{aligned} \quad (25)$$

Applying the Separation Theorem, the value of the coupon at time t ($t < u$) is given by:

$$\begin{aligned} a(r, t; u, s, \Psi_e) &= P(r, t; u) \hat{E}_{r,t} [a(\bar{r}, u; u, s, \Psi_e)] = \\ &= P(r, t; u) \hat{E}_{r,t} \left\{ P(\bar{r}, u, s) \left[\frac{P(\bar{r}, u, v)}{P(\bar{r}, u, T)} \right]^\xi - P(\bar{r}, u, s) \right\} \omega \end{aligned} \quad (26)$$

Using the moment generating function {Jamshidian [1987; p. 16]}, we obtain:

¹¹. See also Longstaff [1990, Equation (3), p. 101].

$$a(r, t; u, s, \Psi_e) = P(r, t; u)F(r, t; u) \quad (27)$$

where:

$$\begin{aligned} F(r, t; u) &= \hat{E}_{r,t} \left\{ P(\tilde{r}, u, s) \left[\frac{P(\tilde{r}, u; v)}{P(\tilde{r}, u; T)} \right]^\xi - P(\tilde{r}, u; s) \right\} \omega = \\ &= \left\{ A_F \left(\frac{1}{1 + zB_F} \right)^{\phi_3} e^{-\left[\frac{4\phi_1^2 w^2 e^{\phi_1(u-t)} B_F}{1 + zB_F} \right]} - \frac{P(r, t; s)}{P(r, t; u)} \right\} \omega \end{aligned} \quad (28)$$

$$A_F = A(u, s) \left[\frac{A(u, v)}{A(u, T)} \right]^\xi \quad (29)$$

$$B_F = B(u, s) + [B(u, v) - B(u, T)]\xi \quad (30)$$

$$w = [2\phi_2(e^{\phi_1(u-t)} - 1) + 2\phi_1]^{-1} \quad (31)$$

$$z = \sigma^2 w (e^{\phi_1(u-t)} - 1) \quad (32)$$

Equation (27) is a simple generalization of the formulas in Castellani, De Felice and Moriconi (1989) and in Sundaresan (1991a, 1991b) for the valuation, respectively, of stochastic zero-coupon bonds, variable-rate payments and Eurodollar forward prices.¹² The formula permits the valuation of floating rate coupons even in the case in which the base yield is forward and related to an interval that is different from the reference period (e.g. the semi-annual equivalent, under compound interest, of an annual yield).¹³

It should be noted that if the interest rate $R(r, t; v, T, \xi, \omega)$ is not referred to a different interval ($\xi = \omega = 1$) and the reference period (v, T) coincides with the residual maturity (u, s) of the security at the time u that its value is determined $u = v$ and $s = T$, we have:¹⁴

$$a(r, t; u, s, \Psi_e) = P(r, t; u) - P(r, t; s) \quad (33)$$

In this case, the current value of the coupon given by (33) can also be obtained discounting the appropriate forward interest rate.¹⁵ In fact:

$$\begin{aligned} a(r, t; u, s, \Psi_e) &= P(r, t; s)R(r, t; s, 1, 1) = \\ &= P(r, t; s) \left[\frac{P(r, t; u)}{P(r, t; s)} - 1 \right] = P(r, t; u) - P(r, t; s) \end{aligned} \quad (34)$$

It should also be noted that at the time the coupon is determined ($t = u$), the value given by equation (27) becomes that of an ordinary fixed rate security:

¹². Specifically, for $u = v$ and $\xi = \omega = 1$, we have $a(r, t; u, s, \Psi_e) = \beta(r, t; T, s, 1) - P(r, t; s)$, where $\beta(r, t; T, s, 1)$ is the value shown in Castellani, De Felice and Moriconi [1989; (37), p. 17] corrected for a misprint [the ratio of $B(T, s)$ to $B(T, T + 1)$ needs to be replaced by their difference]. In addition, for $u = v$ e $\omega = \xi = 1$ we have $a(r, t; u, s, \Psi_e) = V(t, s_N)$, where $V(t, s_N)$ is the value shown in Sundaresan [1991a; Proposition 3, p. 427]. Lastly, for $u = v = s$, $\omega = 100/(T - v)$ and $\xi = 1$, we have $a(r, t; u, s, \Psi_e) = P(r, t; s)[100 - G_t(s)]$, where $G_t(s)$ is the value shown in Sundaresan [1991b; (19), p. 417].

¹³. For securities linked to forward rates, see also Moriconi [1991, p.7].

¹⁴. See also Cox, Ingersoll and Ross (1980) p. 393, Barone and Cesari (1986) pp. 23-25.

¹⁵. See also De Felice and Moriconi [1992; (27), p. 335].

1. In addition to those indicated in the figure, the following values were hypothesized: $\kappa=0,2$ $\mu=0,1$ $\lambda=-0,046$ $\sigma=0,1$ $t=0$ $v=u$ $T=v+1$ $s=T$ $\xi=1/(T-v)$ $\omega=100$.

Figure 2 Elementary coupons and the instantaneous interest rate

$$\begin{aligned}
 a(r, u; u, s, \Psi_e) &= P(r, u; u) F(r, u; u) = \\
 &= [A_F e^{-B_F r} - P(r, u; s)] \omega = \\
 &= \left\{ P(r, u; s) \left[\frac{P(r, u; v)}{P(r, u; T)} \right]^\xi - P(r, u; s) \right\} \omega = \\
 &= P(r, u; s) \left\{ \left[\frac{P(r, u; v)}{P(r, u; T)} \right]^\xi - 1 \right\} \omega = \\
 &= P(r, u; s) \bar{a}
 \end{aligned} \tag{35}$$

where $\bar{a}$ denotes the, known, value of the coupon.

3.3 A sensitivity analysis

The current value of a floating rate coupon is a complex function of the instantaneous interest rate. Positive changes in r reduce the value of the discount factor $P(r, t; u)$ but increase the expected value of the payoff, represented by $(r, t; u)$. The combined effect on $a(r, t; u, s, \Psi_e)$ is generally positive for coupons of relatively short maturity and negative for the others Figure 2. The function $a(r, t; u, s, \Psi_e)$ tends towards a finite value as r tends to zero and towards zero as r tends towards infinity; it is therefore humped for coupons with a shorter maturity, while it decreases monotonically in all other cases.

The effects on the floating rate coupon produced by changes in the model parameters are shown in Figure 3, for three different values of the instantaneous interest rate. The price of this elementary security behaves in a symmetrically opposite way to that of a pure discount bond: it is an increasing (decreasing) function of the speed of adjustment κ depending on whether the instantaneous interest rate is below (above) its long-run expected value, μ , and is an increasing function of μ . The value of the coupon also decreases with increases in the parameter λ , i.e. as the market price of

Figure 3 Elementary coupons and the model parameters

the risk decreases, and with increases in the parameter σ which measures the volatility of the instantaneous interest rate.

Figure 4 shows the effects on the coupon produced by changes in the contractual conditions: the date on which the coupon is calculated (u), the interval between the date on which the coupon is calculated and the date on which the spot rate is observed ($v - u$), the reference period of the base rate ($T - v$) and the interval between the date on which the coupon is calculated and that on which it is paid ($s - u$). The first graph shows that coupons with intermediate coupon calculation dates may have higher values than the others if the term structure has a positive slope ($r = 0,05$ in the example). In general, however, higher values of the variables in question have a negative effect on the value of the coupon. It should be noted, moreover, that in every case an increase in the instantaneous interest rate has a positive effect on the coupons with a shorter maturity and a negative effect on all the others.

3.4 The forward approximation

It was shown earlier that determining the current value of a floating rate coupon by discounting the forward interest rate is only correct in the special case represented by equation (33). Table a7 shows the approximation errors caused by the use of the following formula, which involves discounting the forward interest rate:¹⁶

$$a(r, t; u, s, \Psi_e) \approx P(r, t; s) \left\{ \left[\frac{P(r, t; v)}{P(r, t; T)} \right]^\xi - 1 \right\} \omega \quad (36)$$

¹⁶ This question was raised by Cesari [1992, p. 225].

Figure 4 Elementary coupons and contractual conditions

Overvaluations occur if the coupons are paid before the end of the reference period of the base yield (1 year in the example) and undervaluations in the opposite case. The error is very small in the latter case even if the base yield is a long-term interest rate.¹⁷ The approximation nonetheless gets worse as the parameter σ increases.

4. THE REFERENCE PARAMETER OF CCTs

4.1 Tax aspects

In the following analysis we study the Italian T-bill and CCT markets from the standpoint of an individual investor.¹⁸ In the case of CCTs the tax withheld at source (in settlement of the individual's

¹⁷. See also El Karoui and Geman [1991, p. 60]: "Substantial information about the price and behaviour of floating-rate instruments is given by the forward rates. It is therefore available in the current yield curve. Moreover, for all practical purposes, the correcting term is small".

¹⁸. It has been clearly shown by Rovelli [1991, pp. 21-22] that tax arbitrage leads to corner solutions in which different groups of investors hold only one of the various different tax categories of security. In order to minimize the total tax burden, juridical persons with large interest expenses and overheads (such as banks) should hold only securities taxed at the rate of 12.5 per cent, which provide the highest gross yield. By contrast, juridical persons with negligible interest expenses and overheads (such as insurance companies), since they are not taxed indirectly as a result of not being able to deduct these expenses, should hold only tax-exempt securities. In theory the yield differential between these two categories of security should be equal to the corporate tax rate, which is currently 47.8 per cent. In view of the size of this spread, individuals and mutual funds, which are subject to comparable treatment, should hold only taxed securities, especially those with withholding tax at 6.25 per cent.

liabilities) was deducted from the payments of interest and principal, while for T-bills, on which the tax is levied in advance, we considered the prices inclusive of the withholding tax.

For instance, if we consider T-bills and CCTs with a face value of one and the same residual maturity (of less than six months), the following equilibrium condition holds for individuals:

$$\frac{1}{P_{BOT}} = \frac{1 - \theta(1 - P_0) + \alpha(1 - \theta)}{P_{CCT}} \quad (37)$$

where

P_{BOT} is the price of the T-bills (inclusive of tax);

P_{CCT} is the (*cum-coupon*) price of the CCTs;

P_0 is the base price at issue of the CCTs;

α is the gross coupon of the CCTs;

θ is the tax rate.

Since the prices of the T-bills were modeled inclusive of the tax (P) a problem arises in valuing CCTs with Ψ_6, Ψ_7, Ψ_8 e Ψ_9 , linkages, for which the coupons are determined on the basis of the auction prices exclusive of the tax (Π).

In view of the relationship between the two prices

$$P = \Pi + \theta(1 - \Pi) \quad (38)$$

we have:

$$\Pi = \frac{P - \theta}{1 - \theta} \quad (39)$$

The value of the elementary coupon at maturity is therefore given by:

$$a(r, s; u, s, \Psi_e) = R(r, u; v, T, \xi, \omega) = \left\{ \left[\frac{P(r, u; v) - \theta}{P(r, u; T) - \theta} \right]^\xi - 1 \right\} \omega \quad (40)$$

The relationship of figure (40) has been approximated by an exponential function that, for $\bar{r}(t) = r$ has the same value and the same first and second derivatives.¹⁹ Using the moment generating function, we obtain the valuation formula (b1) shown in the Appendix (Table a8).

4.2 The portfolios of elementary coupons

Turning to the case of a Ψ_m linkage, in which the value of the coupon at maturity s is equal to the arithmetic mean of the d interest rates $R(r_k, u_k; v_k, T_k, \xi_k, \omega_k)$ (for $k = 1, \dots, d$) determined at the times $u_1, \dots, u_d$, relative to the periods $(v_1, T_1), \dots, (v_d, T_d)$ and related to an interval different from the reference periods by means of the coefficients $\xi_1, \dots, \xi_d \in \omega_1, \dots, \omega_d$ we have:

$$a(r, s; u, s, \Psi_m) = \sum_{k=1}^d \frac{R(r, u_k; v_k, T_k, \xi_k, \omega_k)}{d} \quad (41)$$

Since the expected value of a sum is equal to the sum of the expected values, the current value of the coupon is equal to the arithmetic mean of the current values $a(r, t; u_k, s, \Psi_e)$ of the d elementary floating rate coupons that yield the interest rates $R(r_k, u_k; v_k, T_k, \xi_k, \omega_k)$ at maturity s .²⁰

¹⁹. This approximation gives results that are practically equal to those obtained using Taylor's series expansion with five terms.

²⁰. See also Ramaswamy and Sundaresan [1986; p. 259].

$$a(r, t; u, s, \Psi_m) = \sum_{k=1}^d \frac{a(r, t; u_k, s, \Psi_e)}{d} \quad (42)$$

Formula (42) permits the valuation of the floating rate coupons determined as the mean of several base yields.

4.3 The price of risk

The current value of known coupons, the spread and the face value can easily be calculated using the discount factors defined by equation (8) while the formulas (27) or (b1) and (42) permit the determination of the current value of the floating rate coupons with $\Psi_2, \Psi_3, \Psi_4, \Psi_6$ e Ψ_8 , linkages, ignoring the roundings they envisage and putting $\xi = 1$ and $\omega_k = [1/(12\tau_k)]f_k$.

The same formulas also give a good approximation of the current value of the coupons with Ψ_5, Ψ_7 and Ψ_9 linkages²¹. In fact:

$$a(r, s; u, s, \Psi_m) = \sum_{k=1}^d \frac{R(r, u_k; v_k, T_k, \xi, \omega_k)}{d} \approx \left\{ \left[1 + \sum_{k=1}^d \frac{R(r, u_k; v_k, T_k, 1, \omega_k)}{d} \right]^\xi - 1 \right\} \quad (43)$$

Nonetheless, the one-factor Cox, Ingersoll and Ross model gives values, for any combination of $r, \kappa, \mu, \lambda, \sigma$, that are systematically higher than the actual prices at which the various categories of CCT in circulation are traded.

Theoretically, a correctly constructed floating rate security should quote at par at the coupon reset dates in the absence of a spread. With large spreads such as those foreseen for CCTs, the theoretical prices are necessarily above par, even when account is taken of the lags provided for in the linkage mechanism.²² Unexpected increases in the instantaneous interest rate result in a reduction in the fixed components of CCTs (the known coupons, the spread and the face value) and of the floating rate coupons of longer maturity, while those of shorter maturity increase in value. For plausible values of r , the combined effect is never large enough to justify the large discounts observed in the market (Table a9).²³

The main reason why operators quote CCTs below par is that they require a substantial compensation for the risk that the reference parameter (the T-bill yield) will lose its significance. The alternative offered by a T-bill rollover is notably less risky because operators can choose not to renew securities as they mature if the yields they offer are below those available in the market. By contrast, buying CCTs results in operators being locked into a T-bill rollover without being able to get out, except at cost, if the interest rate on T-bills loses its significance.

In practice, regardless of the reference parameter adopted for the linkage, subscribers, and generally the issuer too, are exposed to the uncertainty regarding the future correlation between the reference parameter and the short-term “market” interest rate. An ordinary floating rate security can be considered as a portfolio made up of a “theoretical” floating rate security that periodically provides the “market” rate and a swap in which the issuer undertakes to pay “the fixed component” (the

²¹. To test the goodness of the approximation, we have considered the net yields at auction of the 12-month T-bills issued in the period 1969-91. The average approximation error of $a(r, t; u, s, \Psi_e)$ for $\xi = 0,5$ and $\omega = 1$ was 0.03 basis points and the maximum error 0.9 basis points. In other words the values are very small and disappear in the rounding process.

²². See Barone and Cesari [1986, p. 32]: «Consider, for example, the first 7-year CCT, issued in November 1983, linked to 6-month T-bills with a semi-annual spread of 1 percentage point. ... The existence ... of the foregoing spurious elements resulted in a theoretical valuation of the security equal to 107.87 lire ... There evidently existed a riskiness extraneous to the model that required a discount of around 8.5 per cent (7.87 + 0.75) on the issue price, set equal to 99.25».

²³. The results shown in Table A9 are in line with those published by Cesari [1992; p. 209] and Drudi and Scalia [1992; p. 22]. For evidence to the contrary, see De Felice and Moriconi [1992; pp. 344 and 349].

spread) and receives the “variable component” (the difference between the market rate and the reference parameter).

In the rest of this paper in order to take account of the additional riskiness regarding the future significance of T-bill yields, we assume operators determine the expected values of coupons that are not yet known on the basis of a different risk parameter ($\lambda = \lambda_2$) from the one used to discount the payments that are certain ($\lambda = \lambda_1$).

This led to the definition of the variable

$$\zeta = \lambda_2 - \lambda_1 \quad (44)$$

and to the hypothesis that the elementary coupons are valued on the basis of the following formula:

$$a(r, t; u, s, \Psi_e) = P(r, t; u)F(r, t; u; \zeta) \quad (45)$$

where $P(r, t; u)$ is calculated, using equation (8), on the basis of $(\kappa\mu, \kappa + \lambda_1, \sigma)$ and $F(r, t; u; \zeta)$ is calculated, using equation (27) or (b1), on the basis of $(\kappa\mu, \kappa + \lambda_2, \sigma)$ or, in other words $(\kappa\mu, \kappa + \lambda_1, \sigma, \zeta)$.

The greater the risk of a future loss of T-bill significance, the higher the value of ζ . This variable is primarily an (inverse) proxy of operators' confidence in the Treasury, but it could also reflect changes in the liquidity of the secondary market.²⁴

5. EMPIRICAL TEST

5.1 Data

The size of the CCT secondary market is shown in (Table a10). Specifically, 34 CCTs were traded at the end of 1983 with a face value of 77,250 billion lire, while at the end of 1991 there were 88 with a face value of 439,091 billion. Turnover in these securities rose from 1,780 billion lire in 1983 to 421,444 billion in 1991.

The period chosen for the empirical test (from 30 December 1983 to 21 August 1992) includes 2,171 working days. The data, observed for each day, cover the quotes of all the government securities²⁵ listed on the Milan Stock Exchange and the related turnover. In addition, we have observed the weighted average prices and related turnover of the securities listed on the screen-based market that began operations on 16 May 1988. For the securities quoted on both markets, we have used the screen-based market price, adopting the average of the best bid and offer prices observed at the close when a particular security was not traded. In total, the estimate is based on 261,826 prices and related turnover figures, of which 57.1 per cent refer to CCTs.

Since government securities are quoted *ex-coupon*²⁶ and are traded for cash, with settlement on the third working day following the trade, prices have been calculated inclusive of the interest accrued (*cum-coupon* prices) by adding to the *ex-coupon* prices the share of the current coupon, net of any withholding tax, accruing (on the basis of simple interest and the business year) from the start of entitlement date to the settlement date inclusive

²⁴. To explain the large discounts in the prices of CCTs compared with their par value, Rovelli [1991] considers 6 factors:

- 1) the perception that the reference parameter may be forced below the equilibrium level;
- 2) the pressure of the supply of CCTs in the primary market;
- 3) the imperfections of the secondary market;
- 4) the weakness of the demand;
- 5) the reduction in the current value of the spreads;
- 6) the reduction in the current value of the known coupon.

The parameter ζ may measure the combined effect of the first four factors.

²⁵. Except for Treasury certificates in ECUs, index-linked certificates and Rendita. Thus the estimate covers T-bills, BTPs, CTOs, CCTs, CTSs, fixed rate certificates and Redimibili (including issues to finance school building programmes) and Cassa Depositi e Prestiti securities.

²⁶. Except for the Rendita and the Redimibili, which are quoted *cum-coupon*.

The features of the coupons that are not known are assumed to remain unchanged; i.e. we assumed that the frequency of T-bill auctions will remain the same (twice in a month) and that each coupon can therefore be considered as the mean of four mini-coupons. The maturity of these mini-coupons has been determined on the basis of a forecast of the calendar for T-bill issues.²⁷

5.2 Methodology

The model was estimated using a methodology similar to that reported in Barone and Cuoco [1991, pp. 15-19].²⁸ It was postulated that, for each of the n securities observed at a certain date:

$$\bar{P}_i = P_i(r, t; \mathbf{s}, \mathbf{a}) + u_i \quad i = 1, 2, \dots, n. \quad (46)$$

where $\bar{P}$ represents the actual (*cum-coupon*) price of the i -th security, P_i the theoretical price, which for CCTs is defined by equation (3), and u_i the error term.

Before carrying out the estimate, it was assumed that the variance of the error was inversely proportional to the turnover.²⁹ i.e. that the dispersion of the errors was greater for securities traded in a thin market. Accordingly, in order to render both the members of equation (46) homoschedastic, they were multiplied by the square root of the turnover (expressed in billions of lire).³⁰ The vector $(r, \kappa\mu, \kappa + \lambda_1, \sigma, \zeta)$, which minimizes the sum of the square of the errors, was then estimated on a daily basis using Marquardt's algorithm.³¹

An example of the calculation of the theoretical value of a CCT is given in Table a11.

5.3 Estimates

Table a12 shows some statistics on the estimates obtained for $(r, \kappa\mu, \kappa + \lambda_1, \sigma, \zeta)$. The movements in the instantaneous interest rate are fairly close to those found by Barone, Cuoco and Zautzik [1991] on the basis of listed Italian Treasury bonds Figure 5.

²⁷.As a general rule the date of the settlement (and the maturity) of the T-bills sold at auction is fixed as the 15th and 30th of the month, or if either of these dates is a holiday as the first subsequent working day.

Bids normally have to be delivered to the Bank of Italy on the second working day before the settlement day for end-month auctions and on the third day before the settlement day for mid-month auctions. Bids have to be delivered by midday of the day fixed.

The result of mid-month auctions is announced in the late afternoon of the same day and that of end-month auctions in the early afternoon of the following day.

The securities are listed on the screen-based market the day after the announcement of the result of the auction.

²⁸. For a different approach, see Majnoni and Scalia [1992], who estimated the CIR model on Italian data using the generalized moments method.

²⁹. Thus rejecting the hypothesis that the true equilibrium prices are those at which no trading occurs.

The basis of this hypothesis ("if the prices of securities are in equilibrium, nobody has an interest in trading") ignores the imperfections of the market. The prices of the securities that are not traded may be out of line but arbitrage is rendered unprofitable by transaction costs (if the securities exist only in very small quantities or are not widely held).

³⁰.The weighting is thus different from that used by Barone and Cuoco [1991]: applying the Breusch-Pagan test to the weighted residuals, the null homoschedasticity hypothesis was not rejected for 46 per cent of the data observed.

The weighting adopted made it unnecessary to eliminate the outliers as required in the earlier work

³¹ The calculation program was made subject to the constraint that the ratio between the values of the parameters in successive iterations should not be less than 0.3 or greater than 3.

The maximum number of iterations was fixed at 99 and that of subiterations at 10.

The initial value of the parameter defining the optimal step length was set at 0.1.

The convergence criterion adopted was $\varepsilon = 0.1$.

The initial values of the coefficients $(r, \kappa\mu, \kappa + \lambda_1, \sigma, \zeta)$ were: 0.10553, 0.27570, 0.27157, 28.361 and 0.081291.

The revision of the initialization coefficients was attempted without success: the estimates for consecutive days often remained unchanged, with the algorithm "blocked" on local minima. For a case with the opposite result, see Majnoni and Scalia [1992, p. 22].

Figure 5 The instantaneous and asymptotic rates and the risk-neutral process

By contrast, compared with the earlier results, the long-run asymptotic rate is much less erratic, especially in the last part of the period. This reflects the larger sample, the different weighting criterion and the growth of the secondary market for government securities.

It can also be seen that the drift of the risk-neutral process (7), which adjusts to the dynamic of the instantaneous rate, is positively correlated with the differential between the long-run asymptotic rate and the instantaneous rate.

The daily estimates of the coefficient $\sigma\sqrt{r}$, relative to the stochastic component of the process, show a falling trend that is in line with the changes in the historical volatility of the instantaneous interest rate, as measured by the standard deviation of the daily changes observed in the previous year (253 observations).

Overall, the parameters $\kappa\mu$, $\kappa + \lambda$ and σ were found to be sufficiently stable, in contrast with ζ (Figure 6). Specifically, the estimates of σr are slightly below those obtained by Barone, Cuoco and Zautzik [1991], probably owing to the inclusion in the sample of Italian Treasury option certificates, since their implicit options are particularly sensitive to this parameter.

5.4 Residuals

The mean value of the unweighted residuals is 0.22 for the whole sample and 0.11 for CCTs on their own, with a standard deviation of respectively 1.25 and 0.66. Since the algebraic sum of the weighted residuals is null by construction, a positive mean value of the unweighted residuals indicates that the securities with a thin market tend to be overvalued compared with those that are more heavily traded.

Figure 6 The parameters of the model

In order to highlight any systematic features, the weighted residuals for the whole sample were regressed on seven dummies that identify the securities different from CCTs (T-bills, BTPs, CTSS, CTOs, Treasury fixed rate certificates, Redimibili and Cassa DD.PP. securities). The results of the regression suggest there are regularities in the residuals (Table a13), with signs that are generally not constant over time.

In particular, in 1991 the market appears to have overvalued CCTs and BTPs and to have undervalued CTOs and CTSSs. The low values of the DW statistic indicate that, owing to the existence of positive autocorrelation, the errors tend to maintain the same sign among the securities with similar maturities. The uncorrected R^2 , of the regressions are generally not high

The weighted residuals of the CCTs were then regressed on seven dummies that identify the different types of linkage: $\Psi_3, \dots, \Psi_9$. In the last of the years considered operators appear to have overvalued the Ψ_4 and Ψ_8 , both of which have annual coupons.

The valuation problems arising from tax law have been highlighted by regressing the weighted residuals on two dummies that identify the 6.25 per cent and 12.5 per cent tax regimes. The results show that for individual investors the CCTs subject to the 6.25 per cent withholding tax were considerably undervalued, while the tax-exempt CCTs were overvalued.³²

Lastly, the weighted residuals of the CCTs with the current (Ψ_9) linkage mechanism were regressed on the spread, the residual maturity and the stochastic duration, as defined by equation (16). The market appears not to value the riskiness of these CCTs correctly since it overvalued those with a long stochastic duration. It appears to err in the opposite direction in the case of securities with a larger spread, which were clearly undervalued in 1991.

³². The overvaluation of the tax exempt CCTs fits with the considerations *sub* footnote 16.

1. The differentials refer to 21 August 1992.

Figure 7 Differentials between actual and theoretical prices¹

The differentials between the actual and theoretical prices of the government securities listed on 21 August 1992 are shown in Figure 7 and Figure 8. The same data, together with other information (actual and theoretical *ex-coupon* prices, *cum-coupon* prices, stochastic duration and turnover) are given in Table a14.

On the date in question the BOTs of all maturities, the shorter maturity BTPs, the early issues of CTOs (especially the first) and some other thinly traded securities³³ appear overvalued (Figure 7).

As regards CCTs, a certain degree of systematicness can be seen in the errors of those with Ψ_4 e Ψ_5 linkages, both of which have annual coupons: the securities with a longer stochastic duration, generally those with a recently determined coupon, were overvalued compared with the others, probably as a result of an underestimation of the risk inherent in such securities Figure 8. By contrast, the Ψ_9 linkage securities to be undervalued were those with a shorter residual maturity.

If on the date in question the market had not perceived an extra risk in connection with CCTs ($\zeta = 0$), the prices of these securities would have been 2.2 percentage points higher on average.

The changes in the residuals of a single issue are illustrated in Figure 9, which shows the *ex-coupon* and theoretical prices of the CCT 3/1/97. The small divergences between the two series indicate that the model describes market prices sufficiently well. The figure also shows the stochastic duration of the same security. It increases as the coupon becomes increasingly determined with each

³³. Specifically, fixed rate Treasury certificates issued in exchange for CCTs. The face value of 7 of the 10 listed issues was very small, ranging from 25 million lire to 9.1 billion.

1. The differentials refer to 21 August 1992.

Figure 8 Differentials between actual and theoretical prices¹

auction of the reference T-bills and at the moment the coupon is detached; it then decreases steadily until the first auction contributing to the determination of the next coupon.³⁴

5.5 The forward rates of Italian T-bills implicit in the prices of CCTs

As was shown in section §4.3, the parameter ζ summarizes the extra risk perceived by operators in connection with the particular reference parameter used for the linkage of CCTs. It is by means of this parameter that the expectations implicit in the term structure are corrected to take account of the current or prospective misalignment of the prices of T-bills compared with the theoretical prices for the same maturities.

In the ten years considered the parameter ζ was far from being constant: it increased considerably from the second half of 1987, rising to a peak between 1989 and 1990 and then decreasing again Figure 6. The changes in ζ are not easy to explain and may also reflect pure alarmism or a liquidity crisis in the secondary market for CCTs. For example, the increase in ζ in the second part of 1987 can be attributed to the major recomposition of corporate portfolios in the period. The shift, which was due to changes in tax law,³⁵ was probably too rapid for a relatively small secondary market to handle.

³⁴ A similar representation was put forward by Professor Franco Moriconi in an IBM conference on "Il calcolo scientifico e tecnico nelle imprese, nei servizi, nella ricerca" (Rome, 12-13 November 1991).

³⁵ See, for example, Rovelli [1991, p. 22]: "... after the introduction of the tax, the coupons on old (tax exempt) bonds were indexed to the net-of-tax T-bill rate. Thus, for investors interested in comparing gross yields on different

Figure 9 Prices and duration of the CCT 3/1/97

1. The shaded area measures the difference between the theoretical yields on T-bills calculated on the basis of $\lambda=\lambda_1$ and $\lambda=\lambda_2$: it reflects the adjustment for the risk of T-bill misalignment implicit in the prices of CCTs. The points represent the differentials between the theoretical yields on T-bills ($\lambda=\lambda_1$) and those actually observed.

Figure 10 Actual and theoretical yields of T-bills¹

Apart from such episodes, one variable that could systematically influence the extra risk perceived by operators is the differential between the theoretical and actual yields of 6 and 12-month T-bills at auction. Positive differences, in addition to justifying the correction of CCT prices to take account of the current misalignment of T-bills, could fuel fears about the future significance of the reference parameter used in the linkage mechanism adopted for these securities.

Figure 10 shows the scatter of the differentials between the theoretical and actual yields of T-bills,³⁶ calculated on the basis of the current term structure ($\lambda = \lambda_1$). In recent years the theoretical yields of T-bills are systematically higher than those actually recorded at auction.

A correlation is also noticeable between the degree of T-bill misalignment and the extra risk perceived by operators with respect to CCTs. The latter is represented in Figure 10 by the difference between the theoretical yields on T-bills obtained on the basis of $\lambda = \lambda_1$ and those obtained on the basis of $\lambda = \lambda_2$. The larger the difference, the greater the extra risk perceived by operators, who react by revising downwards their expectations regarding the yields on T-bills.

assets (such as banks) the relative convenience between investing in T-Bills or in CCTs was shifted in favour of the former.”

³⁶ The actual yields were calculated, using compound interest and with reference to the calendar year, on the basis of the weighted average prices at auction plus the withholding tax. The theoretical yields were calculated on the basis of the estimates obtained for day t , where $t + 3$ is the date for the settlement of the T-bills offered.

As can be seen, the perception of a risk of T-bill misalignment, which had decreased considerably in 1984, increases significantly in the second part of 1987, the year in which the returns on these securities began to be systematically below the theoretical values. Consequently, operators lowered their expectations regarding the future values of the reference parameters and sold the CCTs in their portfolios, thereby depressing their prices.

6. CONCLUSIONS

In the first part of the paper the features of CCTs were described, focusing in particular on the risk that operators perceive as being associated with the reference parameter used to link them to T-bills. This risk is different from the credit, or better seignorage, risk that influences the prices of both fixed and floating rate securities. It is a specific risk that derives from the fact that measures capable of reducing the expected interest payments on CCTs are relatively less hard to take. A simple administrative provision (suspending issues of 12-month T-bills) would, in fact, be sufficient to cancel the special immunization capacity of CCTs in a period of rising interest rates.

The article therefore goes on to tackle the question of valuing CCTs, with special reference to the floating rate coupons. Each coupon is broken down into four mini-coupons, corresponding to the four T-bill auctions that determine the coupon's value.

The formula for the valuation of the elementary floating rate coupons was derived, within the framework of the one-factor Cox, Ingersoll and Ross model, by using the Separation Theorem, according to which every financial asset can be valued by discounting the certainty equivalent of the payoff using a discount factor appropriate to the maturity.

The formula adopted constitutes a simple generalization of those that have already appeared in the literature. It permits the valuation of floating rate coupons even in the case in which the interest rate used for the linkage is a forward rate and applied, on a simple or compound basis, over an interval that is different from the reference period.

While the fixed components of CCTs (the known coupons, the spread and the face value) respond in just one way to an increase in the instantaneous interest rate, the floating rate coupons respond differently: those of shorter maturity increase, while those of longer maturity decrease. By contrast, the value of the coupon generally decreases with the lengthening of one of the time measures (the time to the determination date, the forward lag, the interest rate reference period or the difference between the determination and payment dates).

A test was then made to verify whether one of the approximations most commonly used to value floating rate coupons, consisting in discounting the forward value of the base yield (fixed by the issue rules), is usually highly accurate despite its simplicity.

We then derived approximate valuation formulas to take account of the special characteristics of some of the reference parameters adopted for the linkage of CCTs.

Lastly, since the one-factor CIR model is unable to explain CCT prices that are significantly below par, an additional parameter was introduced that measures the risk of misalignment between the base (T-bill) yields and market rates.

The theoretical development of the model is supplemented by an empirical test. The term structure of interest rates was estimated daily for a period of nearly ten years. The term structures, obtained by means of non-linear regressions and covering the whole range of listed government securities (262,000 observations), were biased towards the most widely traded securities by weighting the observations on the basis of turnover. The five estimates obtained for each day (the instantaneous interest rate and four parameters) provide a satisfactory summary of all the information contained in the secondary market for government securities.

In particular, CCT prices contain the market's expectations regarding the future yields on T-bills. These expectations, which operators adjust to take account of the risk of a current or prospective loss of significance, have been extracted. The conclusion this paper reaches is that the perception of a risk of T-bills being misaligned implicit in the depressed prices of CCTs is not unjustified, and is actually confirmed by T-bill auction rates being systematically below those available in the secondary market.

Further analysis of the intertemporal properties of the estimates obtained will make it possible to assess the relevance of the approach adopted for operational purposes.

APPENDIX

TABLE a1 Symbols

Rounding of x to the nearest multiple of y	$\rho(x, y)$	
Normalization of a number with mantissa x and characteristic y	xEy	
Maximum value between x and y	$\max(x, y)$	
Known or certainty equivalent value of x	$\bar{x}$	
Type of linkage	Ψ_h	
Payments $\left\{ \begin{array}{l} \text{gross} \\ \text{minimum gross} \\ \text{net} \end{array} \right.$	$\alpha = \{\alpha_j\}$ $\alpha_{\min} = \{\alpha_{\min}\}$ $a = \{a_j\}$	$j = 1, \dots, m$
Payment dates ¹	$s = \{s_j\}$	$j = 1, \dots, m$
T-bill auctions ² $\left\{ \begin{array}{l} \text{result date} \\ \text{settlement date} \\ \text{maturity date} \\ \text{time to maturity} \\ \text{face value}^3 \\ \text{base price} \left\{ \begin{array}{l} \text{gross} \\ \text{net} \end{array} \right. \\ \text{base yield} \end{array} \right.$	u_{yjk} v_{yjk} T_{yjk} $\left\{ \begin{array}{l} \tau_{yjk} = T_{yjk} - v_{yjk} \\ t_{jk} = \rho(\tau_{yjk}, 1/12) \parallel 2 \end{array} \right.$ $\left\{ \begin{array}{l} Q_{yjk} \\ q_{yjk} \end{array} \right.$ $\left\{ \begin{array}{l} \Pi_{yjk} \\ P_{yjk} \end{array} \right.$ R_{jk}	$t = 3, 6, 12$ $j = 1, \dots, m$ $k = 1, \dots, d$
Tax rate: $\left\{ \begin{array}{l} \text{of T-bills issued in } v_{yjk} \\ \text{of CCTs} \end{array} \right.$	θ_{jk} θ	$j = 1, \dots, m$ $k = 1, \dots, d$
Adjustment factor ⁴	$f_{jk} = \frac{1 - \theta_{jk}}{1 - \theta}$	$j = 1, \dots, m$ $k = 1, \dots, d$
Theoretical price of a pure discount bond	$P(r, t; T)$	
Price of a coupon bond $\left\{ \begin{array}{l} \text{theoretical} \\ \text{actual} \\ \text{base at issue}^5 \end{array} \right.$	$P(r, t; \mathbf{a}, \mathbf{s})$ $\bar{P}$ P_0	
Stochastic duration	D	
Parameters of the model	$\beta = [r, \phi_1, \phi_2, \phi_3, \zeta]$ o $\beta = [r, k\mu, k + \lambda, \sigma, \zeta]$	

¹ The unit of time adopted is the 365-day year.

² T-bills of duration t issued at the k -th auction valid for the determination of the j -th coupon.

³ Q_{yjk} : securities allocated to operators, including the Bank of Italy, of which Q_c in respect of competitive bids and Q_{nc} in respect of non-competitive bids; q_{yjk} : securities allocated to operators, excluding the Bank of Italy.

⁴ The adjustment factor is explicitly provided for in the rules for the CCTs subject to tax at the rate of 6.25 per cent.

⁵ The price for the placement with the public of securities issued at predetermined conditions, auction price for securities sold by auction without the right of subscription and auction base price for securities sold by auction with the right of subscription.

TABLE a2 Floating-rate Treasury Certificates: Determination of the coupons

<i>Type of Linkage¹</i>	<i>Coupon Interval</i>	<i>Reference Period²</i>	<i>Duration of T-Bills (i)</i>	<i>Base Price³</i>
Ψ_1 1-07-1977 1-12-1980	Semi-annual	Quarter that precedes by one month the month in which the date of the start of coupon entitlement falls.	3,6,12	$P_{jk} = \Pi_0$
1-10-1983				
Ψ_2 1-03-1981 1-05-1984				
1-05-1991	“	Two months that precede by one month the month in which the date of the start of coupon entitlement falls.	6	$P_{jk} = \Pi_0 + \theta_{jk}(1 - \Pi_0)$
Ψ_3 1-06-1984 1-12-1984				
1-12-1991				
Ψ_4 1-01-1985 18-09-1986	Annual	“	12	“
1-09-1996				
Ψ_5 15-07-1985 18-03-1986				
18-03-1991	Semi-annual	“	“	“
Ψ_6 1-10-1986 1-08-1987				
1-08-1997				
Ψ_7 20-07-1987 19-08-1987	Annual	“	“	$\Pi_{jk} = \Pi_1$
19-08-1992				
Ψ_8 1-09-1987 1-10-1987				
1-09-1997	Semi-annual	“	“	“
Ψ_9 2-11-1987				
...				
...	...	...	...	...

1 The dates of the first and last issues are shown for each mechanism, together with the last maturity.

2 When T-bills with the reference maturity are not issued in one of the reference months, account is taken only of the yield of the month in which they were issued. In the event that T-bills are not issued in any of the reference months, the base yield is equal to the latest available figure.

(segue) **TABLE a2**

<i>Base Yield</i> <i>(R_{jk})</i>	<i>Reference Parameter</i> <i>(R_j)</i>	<i>Gross Coupon</i> <i>(a_j)</i>	<i>Tax Rate</i> <i>(θ)</i>	<i>Net Coupon</i> <i>(a_j)</i>
$\frac{\sum_t \left(\frac{1}{P_{ijk}} - 1 \right) \frac{12}{t} q_{ijk}}{\sum_t q_{ijk}}$	$\left[\left(1 + \sum_{k=1}^d \frac{R_{jk}}{d} \right)^{0.5} - 1 \right]$	$\max[\rho(R_j, 5E-4), \alpha_{\min}]$	0	$\rho[\alpha_j(1-\theta), 5E-6]$
$\left(\frac{1}{P_{jk}} - 1 \right) \frac{t}{12\tau_{jk}}$	$\sum_{k=1}^d \frac{R_{jk}}{d}$	$\rho(R_j, 5E-4) + \text{spread}$	“	“
“	“	“	“	“
“	“	“	“	“
“	$\left[\left(1 + \sum_{k=1}^d \frac{R_{jk}}{d} \right)^{0.5} - 1 \right]$	“	“	“
$\left(\frac{1}{P_{jk}} - 1 \right) \frac{t}{12\tau_{jk}} f_{jk}$	$\sum_{k=1}^d \frac{R_{jk}}{d}$	“	0.0625	“
“	$\left[\left(1 + \sum_{k=1}^d \frac{R_{jk}}{d} \right)^{0.5} - 1 \right]$	“	“	“
$\left(\frac{1}{P_{jk}} - 1 \right) \frac{t}{12\tau_{jk}}$	$\sum_{k=1}^d \frac{R_{jk}}{d}$	“	0.125	“
“	$\left[\left(1 + \sum_{k=1}^d \frac{R_{jk}}{d} \right)^{0.5} - 1 \right]$	“	“	“

3 Π_0 : allotment price in uniform price auctions (Π_m) or, in the case of competitive bid auctions, the weighted average price of the competitive bids accepted (Π_c). Π_1 : allotment price in uniform price auctions (Π_m) or, in the case of competitive bid auctions, the weighted average price of the competitive bids (Π_c and non-competitive bids (Π_{nc}) accepted $[(\Pi_c Q_c + \Pi_{nc} Q_{nc}) / (Q_c + Q_{nc})]$.

TABLE a3 CCT coupons: Key data

Security: CCT 1.2.93				
Coupon	Coupon Payment Date	T-Bill Auctions ¹		
	(<i>s_j</i>)	Result (<i>u_{jk}</i>)	Settlement (<i>v_{jk}</i>)	Redemption (<i>T_{jk}</i>)
<i>j</i> = 6	01-02-91	10-05-90	15-05-90	15-05-91
		28-05-90	30-05-90	30-05-91
		12-06-90	15-06-90	15-06-91
		27-06-90	29-06-90	28-06-91

¹ Relative to the two months that precede by one month the month in which the date of the start of coupon entitlement falls.

TABLE a4 Determination of CCT coupons: An example

T-BILL AUCTIONS									
Auction Dates	Duration of T-Bills		Tax Rate (θ_{jk})	Bids				Base Price	
	Days	Years (τ_{jk})		Competitive		Non Competitive		$\frac{\Pi_c Q_c + \Pi_{nc} Q_{nc}}{Q_c + Q_{nc}}$	$\Pi_1 + \theta_{jk}(1 - \Pi_1)$
				Q_c	Π_c	Q_{nc}	Π_{nc}		
6-MONTH T-BILLS ($t = 6$)									
15-05-1990	184	0.504109589	0.125	3998.4	0.9417	1.6	0.9422	0.941700200	0.948987675
30-05-1990	184	0.504109589	0.125	10988.9	0.9460	11.1	0.9465	0.946000505	0.952750441
15-06-1990	182	0.498630137	0.125	3996.0	0.9482	4.0	0.9487	0.948200500	0.954675438
28-06-1990	186	0.509589041	0.125	13487.8	0.9470	12.2	0.9475	0.947000452	0.953625395
12-MONTH T-BILLS ($t = 12$)									
15-05-1990	365	1.000000000	0.125	2999.8	0.8860	0.2	0.8865	0.886000033	0.900250029
30-05-1990	365	1.000000000	0.125	6987.7	0.8925	12.3	0.8930	0.892500879	0.905938269
15-06-1990	364	0.997260277	0.125	2497.8	0.8955	2.2	0.8960	0.895500440	0.908562885
28-06-1990	365	1.000000000	0.125	9988.3	0.8945	11.7	0.8950	0.894500585	0.907688012
CCT COUPONS									
Security ¹	Coupon ²	Base Price	Adjustment Factor (f_{jk})	Base Yield (R_{jk})	Reference Parameter (R_j)	Spread	Gross Coupon (α_j)	Tax Rate (θ)	Net Coupon (a_j)
Ψ_3		0.948987675	1	0.053316249	0.049456578	0.0060	0.0555	0	0.055500
1-08-1984	1-08-1990	0.957250441	1	0.049188507					
1-08-1991	1-02-1991	0.954675438	1	0.047606840					
	($j = 14$)	0.953625395	1	0.047714715					
Ψ_4		0.900250029	1	0.110802519	0.104311595	0.0075	0.1120	0	0.112000
1-08-1986	1-08-1990	0.905938269	1	0.103827970					
1-08-1996	1-08-1991	0.908562885	1	0.100915762					
	($j = 5$)	0.907688012	1	0.101700129					
Ψ_5		0.900250029	1	0.110802519	0.050862310	0.0030	0.0540	0	0.054000
18-02-1986	18-08-1990	0.905938269	1	0.103827970					
18-02-1991	18-02-1991	0.908562885	1	0.100915762					
	($j = 10$)	0.907688012	1	0.101700129					
Ψ_6		0.886000033	0.933333333	0.120090254	0.112950153	0.0075	0.1205	0.0625	0.112970
1-08-1987	1-08-1990	0.892500879	0.933333333	0.112417271					
1-08-1997	1-08-1991	0.895500440	0.933333333	0.109213648					
	($j = 4$)	0.894500585	0.933333333	0.110079437					
Ψ_7		0.886000033	0.933333333	0.120090254	0.054964527	0.0030	0.0580	0.0625	0.054375
19-08-1987	19-08-1990	0.892500879	0.933333333	0.112417271					
19-08-1992	19-02-1991	0.895500440	0.933333333	0.109213648					
	($j = 7$)	0.894500585	0.933333333	0.110079437					
Ψ_9		0.886000033	1	0.128668129	0.058781385	0.003	0.062	0.125	0.054250
1-02-1988	1-08-1990	0.892500879	1	0.120447076					
1-02-1993	1-02-1991	0.895500440	1	0.117014623					
	($j = 6$)	0.894500585	1	0.117942254					

¹ The type of linkage and the issue and maturity dates are shown for each security.² The date of the start of coupon entitlement and the payment date are shown for the j -th coupon. The latter is calculated with reference to the issues of T-bills made in the two months that precede by one month the month in which the date of the start of coupon entitlement falls.

TABLE a5 Securities issued by the Treasury: T-bills (billions of lire)

<i>Year</i>	<i>Gross Issues</i>	<i>Issue Discounts</i>	<i>Redemptions</i>	<i>Tax Withheld</i>	<i>Net Issues</i>	<i>Stocks</i>	<i>Redemption Premium</i>
3 MONTHS							
1977	22,650	797	30,850	0	-8,997	3,500	1,172
1978	10,250	276	12,250	0	-2,276	1,500	336
1979	4,050	140	1,800	0	2,110	3,750	46
1980	44,000	1,603	35,000	0	7,398	12,750	1,248
1981	69,139	3,075	56,256	0	9,808	25,633	2,356
1982	83,224	3,641	89,159	0	-9,576	19,698	4,001
1983	57,228	2,319	67,426	0	-12,517	9,500	2,789
1984	31,500	1,106	31,500	0	-1,106	9,500	1,154
1985	38,021	1,228	39,178	0	-2,385	8,343	1,292
1986	44,068	1,212	38,059	29	4,826	14,352	1,116
1987	90,513	2,412	69,016	233	19,319	35,849	1,784
1988	122,313	3,217	124,162	402	-4,664	34,000	3,281
1989	173,953	5,135	162,452	642	7,008	45,501	4,644
1990	175,206	5,012	171,957	626	-1,136	48,750	4,983
1991	200,288	5,900	196,288	738	-1,163	52,750	5,769
6 MONTHS							
1977	28,450	1,977	22,750	0	3,723	13,750	1,739
1978	28,500	1,592	29,500	0	-2,592	12,750	1,774
1979	21,200	1,178	22,450	0	-2,428	11,500	1,206
1980	61,250	4,441	34,000	0	22,809	38,750	2,251
1981	100,745	8,710	85,750	0	6,285	53,745	6,683
1982	136,838	11,645	123,359	0	1,833	67,224	10,970
1983	127,793	10,098	134,650	0	-16,955	60,367	11,035
1984	94,250	6,490	105,617	0	-17,857	49,000	7,791
1985	109,410	6,910	101,925	0	576	56,485	6,653
1986	109,471	5,779	109,259	100	-5,468	56,697	6,603
1987	113,543	5,678	109,596	492	-1,239	60,644	5,208
1988	154,368	7,890	136,918	986	10,546	78,094	7,070
1989	177,387	10,196	163,594	1,275	4,871	91,887	8,768
1990	214,107	12,077	195,387	1,510	8,153	110,607	11,402
1991	225,169	12,949	222,355	1,619	-8,517	113,421	12,577
12 MONTHS							
1977	15,300	2,054	8,000	0	5,246	15,300	1,010
1978	24,000	2,664	15,300	0	6,036	24,000	2,054
1979	33,000	3,642	24,000	0	5,358	33,000	2,664
1980	22,250	2,992	33,000	0	-13,742	22,250	3,642
1981	28,157	4,477	22,250	0	1,430	28,157	2,992
1982	53,217	8,657	28,157	0	16,403	53,217	4,477
1983	81,329	12,431	53,217	0	15,682	81,329	8,657
1984	101,997	13,685	81,329	0	6,983	101,997	12,431
1985	108,849	13,160	101,997	0	-6,308	108,849	13,685
1986	112,326	11,396	108,849	190	-7,729	112,326	13,160
1987	112,055	10,686	112,326	913	-10,044	112,055	11,396
1988	132,729	13,319	112,055	1,665	9,020	132,729	10,686
1989	147,212	16,366	132,729	2,046	163	147,212	13,319
1990	169,883	18,963	147,212	2,370	6,078	169,883	16,366
1991	176,964	19,634	169,883	2,454	-10,099	176,964	18,963

TABLE a6 Securities issued by the Treasury: CCTs (billions of lire)

<i>Year</i>	<i>Gross Issues</i>	<i>Issue Discounts</i>	<i>Redemptions</i>	<i>Gross Coupons</i>			<i>Tax Withheld</i>	<i>Net Issues</i>	<i>Stocks</i>	<i>Reimbursement Premium</i>
				<i>Actual</i>	<i>Theor.¹</i>	<i>Diff.</i>				
1977	5,000	0	0	0	0	0	0	5,000	5,000	0
1978	10,184	0	0	1,058	1,058	0	0	9,125	15,184	0
1979	11,750	26	5,000	2,160	2,160	0	0	4,564	21,934	0
1980	16,430	74	10,184	3,245	3,401	-156	0	2,927	28,180	0
1981	9,200	134	6,250	4,673	5,074	-401	0	-1,857	31,130	12
1982	47,250	873	19,000	6,608	6,837	-229	0	20,770	59,380	64
1983	74,650	868	7,130	14,014	13,916	98	0	52,638	126,900	108
1984	95,250	262	38,250	25,326	24,816	510	0	31,413	183,900	643
1985	97,210	1,543	17,100	27,456	27,333	123	0	51,111	264,010	250
1986	79,255	571	25,150	38,514	37,680	834	0	15,020	318,115	359
1987	55,480	533	19,680	36,958	35,839	1,119	163	-1,528	353,915	224
1988	27,350	198	35,000	37,980	38,312	-332	718	-45,110	346,265	285
1989	21,300	384	0	38,486	38,610	-124	1,111	-16,459	367,565	0
1990	75,538	1,920	20,017	45,879	46,780	-901	1,733	9,455	423,086	88
1991	99,000	3,285	88,275	52,098	51,890	208	3,058	-41,600	433,811	244

1. The theoretical coupons were calculated, other conditions (first coupon, spread, tax treatment, minimum coupon) being equal, on the assumption that the CCTs with semi-annual (annual) coupons are linked to the yield on 6 (12) month T-bills observed at the auction immediately preceding the date of the start of coupon entitlement.

TABLE a7 Theoretical value of the elementary coupons

EQUATION (27): $a = P(r, t, u) F(r, t, u)$											
			$u = v = 1$			$u = v = 5$			$u = v = 10$		
			r			r			r		
			0,05	0,125	0,20	0,05	0,125	0,20	0,05	0,125	0,20
$\sigma = 0,05$	$s = u$	$T - v$									
		1	6.4912	11.8412	16.7750	6.8761	7.1813	7.0337	4.9039	3.8141	2.9304
		3	7.3497	11.8634	15.9840	7.2201	7.1900	6.8305	5.0044	3.8218	2.8925
		5	8.0470	11.8635	15.3163	7.4921	7.1865	6.6547	5.0795	3.8226	2.8573
		7	8.6162	11.8524	14.7564	7.7097	7.1772	6.5047	5.1368	3.8198	2.8259
	$s = T$	10	9.2857	11.8280	14.0836	7.9609	7.1595	6.3218	5.2000	3.8131	2.7863
		1	6.0651	10.4220	13.9102	6.2275	6.2973	5.9740	4.3617	3.3438	2.5326
		3	5.8556	8.0982	9.3540	5.3166	4.8727	4.2639	3.5193	2.5904	1.8900
		5	5.3344	6.2910	6.5024	4.4415	3.7811	3.1107	2.8201	2.0131	1.4278
		7	4.6685	4.8894	4.6389	3.6513	2.9397	2.3067	2.2474	1.5675	1.0880
$\sigma = 0,15$	$s = u$	10	3.6333	3.3547	2.8978	2.6639	2.0201	1.5055	1.5865	1.0793	0.7319
		1	6.4715	11.8013	16.7173	6.5316	6.7362	6.5754	4.5929	3.5966	2.8001
		3	7.1975	11.5876	15.5909	6.7964	6.6843	6.3221	4.6945	3.6236	2.7856
		5	7.6977	11.2784	14.5093	6.9310	6.5663	6.0480	4.7224	3.6039	2.7425
		7	8.0491	10.9658	13.5724	6.9978	6.4357	5.7980	4.7177	3.5681	2.6931
	$s = T$	10	8.4032	10.5647	12.4713	7.0403	6.2619	5.4955	4.6914	3.5125	2.6264
		1	5.9760	10.2501	13.6789	5.7667	5.7540	5.4485	3.9972	3.0950	2.3837
		3	5.6415	7.7759	9.0025	4.8359	4.3900	3.8544	3.2356	2.4326	1.8232
		5	5.0861	6.0015	6.2604	4.0668	3.4601	2.8804	2.6621	1.9630	1.4447
		7	4.4566	4.7098	4.5540	3.4152	2.7793	2.2287	2.2045	1.6047	1.1666
10	3.5405	3.3532	3.0003	2.6145	2.0387	1.5757	1.6664	1.1983	0.8611		
APPROXIMATION ERROR: (27) – (36)											
			$u = v = 1$			$u = v = 5$			$u = v = 10$		
			r			r			r		
			0,05	0,125	0,20	0,05	0,125	0,20	0,05	0,125	0,20
$\sigma = 0,5$	$s = u$	$T - v$									
		1	0.0105	0.0231	0.0355	0.0323	0.0409	0.0447	0.0298	0.0259	0.0217
		3	0.0156	0.0343	0.0523	0.0479	0.0605	0.0657	0.0441	0.0383	0.0319
		5	0.0177	0.0387	0.0585	0.0541	0.0679	0.0735	0.0496	0.0429	0.0357
		7	0.0181	0.0393	0.0591	0.0550	0.0688	0.0743	0.0503	0.0435	0.0361
	$s = T$	10	0.0172	0.0369	0.0552	0.0518	0.0646	0.0694	0.0472	0.0407	0.0338
		1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		3	-0.0062	-0.0117	-0.0153	-0.0177	-0.0205	-0.0206	-0.0155	-0.0130	-0.0104
		5	-0.0078	-0.0137	-0.0166	-0.0214	-0.0239	-0.0230	-0.0184	-0.0151	-0.0119
		7	-0.0074	-0.0122	-0.0140	-0.0195	-0.0211	-0.0198	-0.0165	-0.0134	-0.0104
$\sigma = 0,15$	$s = u$	10	-0.0055	-0.0086	-0.0093	-0.0141	-0.0149	-0.0135	-0.0117	-0.0094	-0.0072
		1	0.0925	0.2042	0.3137	0.2484	0.3079	0.3342	0.2043	0.1734	0.1441
		3	0.1309	0.2861	0.4352	0.3403	0.4182	0.4509	0.2773	0.2343	0.1940
		5	0.1381	0.2995	0.4518	0.3515	0.4293	0.4606	0.2847	0.2398	0.1981
		7	0.1309	0.2819	0.4225	0.3284	0.3992	0.4268	0.2649	0.2227	0.1836
	$s = T$	10	0.1120	0.2393	0.3560	0.2773	0.3355	0.3572	0.2228	0.1870	0.1539
		1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		3	-0.0515	-0.0971	-0.1276	-0.1218	-0.1398	-0.1412	-0.0962	-0.0795	-0.0644
		5	-0.0607	-0.1072	-0.1320	-0.1361	-0.1515	-0.1488	-0.1058	-0.0864	-0.0693
		7	-0.0538	-0.0912	-0.1076	-0.1168	-0.1276	-0.1232	-0.0899	-0.0730	-0.0582
10	-0.0379	-0.0619	-0.0704	-0.0800	-0.0860	-0.0818	-0.0611	-0.0493	-0.0391		

The values of the model parameters considered are as follows: $\kappa\theta = 0.02$, $\kappa + \lambda = 0.154$, $\sigma = 0.05, 0.15$, $r = 0.05, 0.125, 0.20$ (respectively a rising, humped and falling term structure of interest rates). It was also assumed that the base yield is represented by a spot interest rate at 1, 3, 5, 7 and 10 years, observed at the time $u = 1, 5, 10$, annualized [$\zeta = 1/(T - v)$] and expressed as a percentage ($\omega = 100$). Lastly, it is assumed that the coupons are paid on the date the base yield is determined ($s = u$) or matures ($s = T$). The value of the elementary coupons was calculated on the basis of equation (27) and compared with the approximation obtained by using equation (36).

TABLE a8 Theoretical value of elementary coupons: An analytical approximation

$$a(r, t; u, s, \Psi_f) = P(r, t; u)F(r, t; u) \quad (\text{b1})$$

where

$$\begin{aligned} F(r, t; u) &= \hat{E}_{r,t} \left\{ P(\tilde{r}; u, s) \left[\frac{P(\tilde{r}, u; v) - \theta}{P(\tilde{r}, u; T) - \theta} \right]^\xi - P(\tilde{r}, u; s) \right\} \omega \approx \\ &\approx \left\{ A_F \left(\frac{1}{1 + zB_F} \right)^{\phi_3} e^{-\left[\frac{4\phi_1^2 w^2 e^{\phi_1(u-t)} B_F}{1 + zB_F} \right] r} - \frac{P(r, t; s)}{P(r, t; u)} h_2 \right\} \omega \end{aligned} \quad (\text{b2})$$

e

$$A_F = A(u, s)h_0 \quad (\text{b3})$$

$$B_F = B(u, s) + h_1 \quad (\text{b4})$$

$$w = \left[2\phi_2 \left(e^{\phi_1(u-t)} - 1 \right) + 2\phi_1 \right]^{-1} \quad (\text{b5})$$

$$z = \sigma^2 w \left[e^{\phi_1(u-t)} - 1 \right] \quad (\text{b6})$$

$$h_0 = \frac{g_1}{h_1 e^{h_1 r}} \quad (\text{b7})$$

$$h_1 = \frac{g_2}{g_1} \quad (\text{b8})$$

$$h_2 = 1 - g_0 + h_0 e^{h_1 r} \quad (\text{b9})$$

$$g_0 = \left[\frac{P(r, u; v) - \theta}{P(r, u; T) - \theta} \right]^\xi \quad (\text{b10})$$

$$g_1 = \xi g_0 (f_2 - f_1) \quad (\text{b11})$$

$$g_2 = \frac{g_1^2}{g_0} - \xi g_0 \left[\frac{f_1 B(u, v) \theta}{P(r, u; v) - \theta} - \frac{f_2 B(u, T) \theta}{P(r, u; T) - \theta} \right] \quad (\text{b12})$$

$$f_1 = \frac{B(u, v) P(r, u; v)}{P(r, u; v) - \theta} \quad (\text{b13})$$

$$f_2 = \frac{B(u, T) P(r, u; T)}{P(r, u; T) - \theta} \quad (\text{b14})$$

AT ISSUE											
		$r = 0.05$			$r = 0.125$			$r = 0.20$			
		μ			μ			μ			
		κ	0.04	0.11	0.18	0.04	0.11	0.18	0.04	0.11	0.18
$\sigma = 0.05$	$\lambda = 0$	0.1	7.8125	7.7027	7.6340	5.5281	5.5523	5.5992	3.8804	3.9838	4.0989
		0.3	7.8611	7.4403	7.2536	5.7724	5.5658	5.5373	4.0309	3.9874	4.0786
		0.5	7.8982	7.2671	7.0217	6.0232	5.5858	5.4755	4.3516	4.0770	4.0797
	$\lambda = -0.05$	0.1	7.7806	7.7273	7.7089	5.5395	5.6220	5.7202	4.0045	4.1582	4.3174
		0.3	7.7977	7.4289	7.3027	5.6770	5.5466	5.5917	3.9524	3.9958	4.1651
		0.5	7.8393	7.2408	7.0498	5.9313	5.5473	5.5020	4.2508	4.0437	4.1179
	$\lambda = -0.10$	0.1	7.7982	7.8157	7.8561	5.6452	5.7890	5.9388	4.2509	4.4508	4.6496
		0.3	7.7422	7.4435	7.3899	5.5994	5.5629	5.6922	3.9082	4.0528	4.3079
		0.5	7.7798	7.2257	7.0992	5.8392	5.5226	5.5529	4.1540	4.0291	4.1849
$\sigma = 0.15$	$\lambda = 0$	0.1	7.9626	7.8595	7.7915	5.7487	5.7591	5.7908	4.0940	4.1764	4.2713
		0.3	7.9422	7.5344	7.3443	5.8938	5.6813	5.6377	4.1679	4.1066	4.1760
		0.5	7.9407	7.3194	7.0690	6.0835	5.6463	5.5257	4.4217	4.1401	4.1286
	$\lambda = -0.05$	0.1	7.9510	7.8998	7.8778	5.7716	5.8347	5.9129	4.2087	4.3380	4.4744
		0.3	7.8957	7.5376	7.4033	5.8206	5.6773	5.7007	4.1094	4.1266	4.2672
		0.5	7.8906	7.3016	7.1028	6.0036	5.6170	5.5577	4.3337	4.1155	4.1714
	$\lambda = -0.10$	0.1	7.9795	7.9938	8.0263	5.8699	5.9906	6.1179	4.4240	4.5996	4.7758
		0.3	7.8593	7.5668	7.4988	5.7662	5.7072	5.8070	4.0833	4.1914	4.4103
		0.5	7.8416	7.2958	7.1578	5.9254	5.6022	5.6137	4.2516	4.1098	4.2420
THREE MONTHS AFTER											
		$r = 0.05$			$r = 0.125$			$r = 0.20$			
		μ			μ			μ			
		κ	0.04	0.11	0.18	0.04	0.11	0.18	0.04	0.11	0.18
$\sigma = 0.05$	$\lambda = 0$	0.1	5.1337	5.1441	5.1779	3.2759	3.3713	3.4789	1.8597	2.0035	2.1529
		0.3	5.4334	5.2170	5.1769	3.5455	3.4943	3.5763	1.9189	1.9918	2.1646
		0.5	5.7115	5.2845	5.1759	3.9257	3.6587	3.6613	2.2915	2.1579	2.2522
	$\lambda = -0.05$	0.1	5.1067	5.1745	5.2589	3.3454	3.4919	3.6444	2.0540	2.2390	2.4246
		0.3	5.3285	5.1812	5.2096	3.4453	3.4763	3.6331	1.8573	2.0163	2.2642
		0.5	5.6133	5.2339	5.1859	3.8125	3.6083	3.6789	2.1823	2.1211	2.2875
	$\lambda = -0.10$	0.1	5.1590	5.2902	5.4277	3.5269	3.7222	3.9165	2.3755	2.5954	2.8115
		0.3	5.2374	5.1766	5.2843	3.3737	3.5015	3.7413	1.8412	2.0967	2.4249
		0.5	5.5143	5.1957	5.2183	3.7018	3.5743				

¹ The security considered is a 10-year CCT with a semi-annual coupon, Ψ_g linkage, and a spread equal to a semi-annual 0.5 per cent. The figures in the second part of the table were determined by assuming that three months after the date on which the first coupon was determined the instantaneous interest rate had risen by 5 percentage points. The *cum*-coupon prices were then calculated and the *ex*-coupon prices derived by deducting the accrued interest.

TABLE a10 Size of the secondary market for CCTs ¹

MILAN STOCK EXCHANGE			
<i>Year</i>	<i>Number of Securities¹</i>	<i>Face Value¹ (Bn.)</i>	<i>Turnover (Bn.)</i>
1983	34	77,250	-
1984	48	138,400	1,780
1985	50	183,800	1,973
1986	55	237,685	3,698
1987	73	347,215	8,403
1988	82	347,845	13,130
1989	90	367,845	11,076
1990	86	384,328	20,325
1991	82	385,591	18,834
SCREEN-BASED MARKET			
<i>Year</i>	<i>Number of Securities¹</i>	<i>Face Value¹ (Bn.)</i>	<i>Turnover (Bn.)</i>
1983	-	-	-
1984	-	-	-
1985	-	-	-
1986	-	-	-
1987	-	-	-
1988	20	135,996	30,717
1989	19	135,449	38,203
1990	27	207,987	229,790
1991	33	272,787	402,610
TOTAL			
<i>Year</i>	<i>Number of Securities¹</i>	<i>Face Value¹ (Bn.)</i>	<i>Turnover (Bn.)</i>
1983	34	77,250	-
1984	48	138,400	1,780
1985	50	183,800	1,973
1986	55	237,685	3,698
1987	73	347,215	8,403
1988	82	347,845	43,847
1989	90	367,845	49,279
1990	91	428,366	250,115
1991	88	439,091	421,444

¹ At the end of the period

TABLE a11 Theoretical value of CCTs: An example

SECURITY: CCT 1.2.93											
Code	Type of Linkage	Tax Rate	Spread	Issue Base Price	Listing Date	Settlement Date	Ex-Coupon Price	Cum-Coupon Price	β		
13011	Ψ_9	12.5	0.3	99.25	26-06-1990	29-06-1990	99.35	104.24	$\left\{ \begin{array}{c} 0.09618 \\ 0.25784 \\ 0.25223 \\ 23.86182 \\ 0.0903 \end{array} \right\}$		
VALUE OF THE COUPONS NOT YET KNOWN ¹											
Coupon	Payment Date	T-Bill Auction: ² Planned Dates			Maturities				Mini-Coupons ³		Coupons ⁴
		Result	Settlement	Redemption	s	u	v	T	Known	Unknown	
j = 6	01-02-1991	10-05-1990	15-05-1990	15-05-1991	0.6027	-	-	-	5.877229	-	5.538046
		28-05-1990	30-05-1990	30-05-1991		-	-	-	5.512072	-	
		12-06-1990	15-06-1990	15-06-1991		-	-	-	5.359611	-	
		27-06-1990	29-06-1990	28-06-1991		0.0027	0.0082	1.0055	-	5.403273	
j = 7	01-08-1991	12-11-1990	15-11-1990	15-11-1991	1.0986	0.3808	0.3890	1.3890	-	5.167228	5.169258
		28-11-1990	30-11-1990	29-11-1991		0.4247	0.4301	1.4274	-	5.167802	
		11-12-1990	14-12-1990	16-12-1991		0.4603	0.4685	1.4740	-	5.171024	
		27-12-1990	31-12-1990	30-12-1991		0.5041	0.5151	1.5123	-	5.170978	
j = 8	03-02-1992	10-05-1991	15-05-1991	15-05-1992	1.6082	0.8712	0.8849	1.8877	-	4.925586	4.925624
		28-05-1991	30-05-1991	29-05-1992		0.9205	0.9260	1.9260	-	4.924437	
		11-06-1991	14-06-1991	15-06-1992		0.9589	0.9671	1.9726	-	4.926045	
		26-06-1991	28-06-1991	30-06-1992		1.0000	1.0055	2.0137	-	4.926428	
j = 9	03-08-1992	12-11-1991	15-11-1991	16-11-1992	2.1068	1.3808	1.3890	2.3945	-	4.685892	4.684987
		27-11-1991	29-11-1991	30-11-1992		1.4219	1.4274	2.4329	-	4.685014	
		11-12-1991	16-12-1991	15-12-1992		1.4603	1.4740	2.4740	-	4.684253	
		23-12-1991	30-12-1991	30-12-1992		1.4932	1.5123	2.5151	-	4.684788	
j = 10	01-02-1993	12-05-1992	15-05-1992	14-05-1993	2.6055	1.8795	1.8877	2.8849	-	4.445884	4.444723
		27-05-1992	29-05-1992	31-05-1993		1.9205	1.9260	2.9315	-	4.445823	
		10-06-1992	15-06-1992	15-06-1993		1.9589	1.9726	2.9726	-	4.444437	
		26-06-1992	30-06-1992	30-06-1993		2.0027	2.0137	3.0137	-	4.442747	
VALUE OF THE SECURITY											
Payment Date	Maturity ⁶	Discount Factor	Valori a Scadenza ⁵				Valori Correnti				
			Known Coupons	Unknown Coupons	Spread	Principal	Known Coupons	Unknown Coupons	Spread	Principal	Total
01-08-1990	0.0904	99.1301	5.9065	-	-	-	5.8551	-	-	-	-
01-02-1991	0.5945	94.2834	-	5.1625	0.2625	-	-	4.8633	0.2473	-	-
01-08-1991	1.0904	89.5568	-	5.0750	0.2625	-	-	4.5411	0.2349	-	-
03-02-1992	1.6000	84.7841	-	5.0750	0.2625	-	-	4.2989	0.2224	-	-
03-08-1992	2.0986	80.2287	-	5.1185	0.2625	-	-	4.1029	0.2104	-	-
01-02-1993	2.5973	75.8107	-	5.1185	0.2625	99.9060	-	3.8769	0.1988	75.7395	104.3900
							5.8551	21.6831	1.1138	75.7395	104.3900

¹ Current value of the coupons not yet known, no spread and gross of withholding tax.² Relative to the two months that precede by one month the month in which the date of the start of coupon entitlement falls³ The value of the mini-coupons that are not yet known was determined using the formulas (b1) and (42), with $\xi = 0,5$ and $\omega_k = [t/(12\tau_k)]$, on the basis of the values of β reported above.⁴ Calculated as the arithmetic mean of the mini-coupons.⁵ Net of withholding tax and then rounded.⁶ The time, expressed in years, between the settlement and payment dates.

TABLE a12 Estimates of the CIR model

r						R_e					
Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.	Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.
1984	0.13466	0.14997	0.16446	0.04621	0.30811	1984	0.04759	0.11172	0.16168	0.23423	2.09660
1985	0.11744	0.13202	0.14486	0.06398	0.48465	1985	0.06553	0.11121	0.15889	0.27758	2.49600
1986	0.08969	0.11773	0.14622	0.07728	0.65646	1986	0.03862	0.08344	0.14630	0.30133	3.61148
1987	0.09153	0.10125	0.11306	0.03432	0.33893	1987	0.06966	0.09850	0.12586	0.09287	0.94286
1988	0.08151	0.09331	0.10785	0.04644	0.49770	1988	0.09493	0.12505	0.16601	0.12742	1.01896
1989	0.08971	0.10440	0.11736	0.04004	0.38358	1989	0.11284	0.13324	0.15405	0.09399	0.70539
1990	0.09062	0.10342	0.11316	0.03012	0.29129	1990	0.12135	0.13749	0.15701	0.06342	0.46131
1991	0.10144	0.10766	0.11344	0.01144	0.10623	1991	0.10154	0.11233	0.13224	0.01246	0.11093
Tot.	0.08151	0.11390	0.16446	0.04632	0.40668	Tot.	0.03862	0.11296	0.16601	0.17376	1.53826

$\kappa\mu - (\kappa + \lambda)r$						$\sigma\sqrt{r}$					
Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.	Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.
1984	-0.02839	-0.00975	0.00677	0.00490	-0.50266	1984	0.01407	0.01865	0.02109	0.00084	0.04494
1985	-0.02076	-0.00512	0.01134	0.00609	-1.18935	1985	0.01467	0.01708	0.01959	0.00096	0.05618
1986	-0.02592	-0.00888	0.00863	0.00693	-0.78008	1986	0.01315	0.01550	0.01837	0.00134	0.08636
1987	-0.01022	-0.00037	0.00796	0.00357	-9.58517	1987	0.01333	0.01428	0.01563	0.00068	0.04767
1988	-0.00231	0.00901	0.02277	0.00393	0.43646	1988	0.01318	0.01455	0.01543	0.00051	0.03493
1989	-0.00028	0.00829	0.01668	0.00423	0.51026	1989	0.01506	0.01587	0.01655	0.00025	0.01603
1990	0.00269	0.00971	0.01644	0.00219	0.22510	1990	0.01456	0.01589	0.01653	0.00045	0.02845
1991	-0.00198	0.00170	0.00731	0.00216	1.27364	1991	0.01454	0.01528	0.01631	0.00047	0.03047
Tot.	-0.02839	0.00020	0.02277	0.00854	41.79754	Tot.	0.01315	0.01584	0.02109	0.00149	0.09433

$\kappa\mu$						$\kappa + \lambda$					
Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.	Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.
1984	0.01297	0.03034	0.04383	0.00414	0.13635	1984	0.26562	0.26734	0.27011	0.00055	0.00207
1985	0.01782	0.03021	0.04308	0.00498	0.16479	1985	0.26580	0.26758	0.26939	0.00072	0.00267
1986	0.01051	0.02268	0.03971	0.00619	0.27282	1986	0.26642	0.26806	0.26989	0.00060	0.00225
1987	0.01895	0.02677	0.03417	0.00366	0.13669	1987	0.26714	0.26808	0.26890	0.00043	0.00159
1988	0.02581	0.03395	0.04504	0.00293	0.08636	1988	0.26652	0.26735	0.26839	0.00035	0.00129
1989	0.03065	0.03616	0.04177	0.00253	0.06992	1989	0.26613	0.26696	0.26787	0.00039	0.00145
1990	0.03296	0.03731	0.04258	0.00141	0.03787	1990	0.26628	0.26687	0.26753	0.00019	0.00072
1991	0.02760	0.03051	0.03589	0.00222	0.07278	1991	0.26694	0.26765	0.26806	0.00030	0.00112
Tot.	0.01051	0.03068	0.04504	0.00578	0.18826	Tot.	0.26562	0.26754	0.27011	0.00063	0.00235

σ						ζ					
Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.	Year	Minimum	Mean	Maximum	Standard Deviation	Coeff. Var.
1984	0.03595	0.04819	0.05499	0.00220	0.04559	1984	0.07348	0.11694	0.15644	0.02502	0.21394
1985	0.03998	0.04705	0.05377	0.00278	0.05900	1985	0.03881	0.05076	0.07807	0.00640	0.12616
1986	0.03775	0.04519	0.05128	0.00245	0.05417	1986	0.04369	0.06167	0.08922	0.00872	0.14141
1987	0.04149	0.04487	0.04874	0.00169	0.03775	1987	0.04273	0.06313	0.10203	0.01580	0.25023
1988	0.04335	0.04766	0.05091	0.00132	0.02762	1988	0.07216	0.10066	0.12853	0.01193	0.11855
1989	0.04591	0.04920	0.05210	0.00142	0.02892	1989	0.10298	0.12102	0.13934	0.00620	0.05122
1990	0.04698	0.04943	0.05145	0.00069	0.01397	1990	0.08071	0.10895	0.13583	0.01377	0.12641
1991	0.04496	0.04656	0.04922	0.00114	0.02454	1991	0.05118	0.07499	0.10276	0.01212	0.16167
Tot.	0.03595	0.04707	0.05499	0.00243	0.05156	Tot.	0.03881	0.08627	0.15644	0.02867	0.33234

TABLE a13 Analysis of residuals

		CATEGORIES OF SECURITIES ¹							
		1984	1985	1986	1987	1988	1989	1990	1991
CCT	β_0	<u>-0.1393</u>	<u>0.1046</u>	<u>-0.9315</u>	-0.0375	<u>0.1052</u>	0.3541	2.4346	1.1203
BOT	β_1	-	-	-	-	-	<u>0.5635</u>	<u>-1.9235</u>	<u>-0.4824</u>
BTP	β_2	<u>0.5231</u>	<u>0.1635</u>	<u>2.4781</u>	<u>0.8484</u>	1.0622	<u>0.9103</u>	<u>-1.9980</u>	<u>0.5038</u>
CTO	β_3	-	-	-	-	-	<u>-0.2722</u>	<u>-2.9504</u>	<u>-1.7529</u>
CTS	β_4	-	-	-	-	<u>-1.8131</u>	<u>-4.3374</u>	<u>-5.0265</u>	<u>-1.8410</u>
CTP	β_5	<u>0.3089</u>	<u>0.5503</u>	-	0.0550	<u>0.5811</u>	<u>0.1159</u>	<u>-2.0538</u>	<u>-0.5977</u>
RED	β_6	<u>-0.4468</u>	<u>-0.4212</u>	-0.0661	-	-0.0943	<u>-0.3219</u>	<u>-2.5591</u>	<u>-1.6045</u>
CDP	β_7	<u>-0.6337</u>	0.0950	<u>1.0214</u>	-	-0.0059	-0.2951	<u>-2.7880</u>	<u>-1.2811</u>
	R^2	0.0194	0.0067	0.0776	0.0213	0.0316	0.0810	0.1068	0.0678
	DW	1.4612	1.5332	1.0349	1.2512	1.4277	1.6396	1.4166	1.6303

¹ The weighted residuals of all the securities in the sample were regressed on 7 dummies that identify the securities different from CCTs. The abbreviations CTP, RED and CDP refer respectively to Treasury fixed rate certificates, Redimibili and Cassa Depositi e Prestiti securities.

		CCT: TYPES OF LINKAGES ²							
		1984	1985	1986	1987	1988	1989	1990	1991
Ψ_2	β_0	<u>-0.1498</u>	<u>-0.0923</u>	<u>-1.1919</u>	<u>0.2740</u>	<u>-0.4872</u>	<u>-1.6404</u>	-0.0413	0.0843
Ψ_3	β_1	<u>4.0669</u>	<u>1.4162</u>	<u>-1.5534</u>	0.0934	<u>2.0866</u>	<u>1.6334</u>	<u>0.3417</u>	<u>0.7131</u>
Ψ_4	β_2	-	<u>-0.4442</u>	<u>3.0097</u>	<u>-0.2544</u>	-0.1454	<u>1.3686</u>	<u>5.4147</u>	<u>2.8071</u>
Ψ_5	β_3	-	-	<u>0.2556</u>	<u>-1.3625</u>	<u>0.7021</u>	<u>1.8919</u>	<u>0.5176</u>	-0.0090
Ψ_6	β_4	-	-	-	<u>-0.3646</u>	<u>1.3996</u>	<u>2.7001</u>	<u>-0.6807</u>	<u>-1.0459</u>
Ψ_7	β_5	-	-	-	-	<u>0.5510</u>	<u>0.4371</u>	<u>2.1395</u>	<u>1.7081</u>
Ψ_8	β_6	-	-	-	-	<u>5.0016</u>	<u>10.1887</u>	<u>7.2897</u>	<u>2.3418</u>
Ψ_9	β_7	-	-	-	<u>-0.2366</u>	<u>0.7542</u>	<u>2.7970</u>	<u>2.4550</u>	<u>0.4063</u>
	R^2	0.0120	0.0559	0.1503	0.0321	0.0696	0.2150	0.3180	0.2285
	DW	1.3213	1.1834	0.8951	1.1072	1.3385	1.5677	1.3321	1.2554

² The weighted residuals of the CCTs were regressed on 7 dummies that identify the securities belonging to the different types of linkage.

		CCT: TAX REGIMES ³							
		1984	1985	1986	1987	1988	1989	1990	1991
EXEMPT	β_0	<u>-0.1393</u>	<u>0.1046</u>	<u>-0.9315</u>	-0.0321	<u>-0.1757</u>	<u>-0.3274</u>	<u>3.2083</u>	<u>2.5194</u>
6,25%	β_1	-	-	-	-0.0584	<u>0.9887</u>	<u>1.1208</u>	<u>-3.5986</u>	<u>-3.1571</u>
12,50%	β_2	-	-	-	0.0696	<u>1.0997</u>	<u>2.3876</u>	<u>-0.3991</u>	<u>-1.9210</u>
	R^2	0.0000	0.0000	0.0000	0.0000	0.0173	0.0824	0.0981	0.1698
	DW	1.3219	1.1868	0.7207	1.0375	1.2871	1.4497	1.1207	1.2507

³ The weighted residuals of the CCTs were regressed on 2 dummies that identify the securities taxed at 6.25 per cent and those taxed at 12.5 per cent.

		Ψ_9 TYPE CCT: SPREAD, TIME TO MATURITY AND STOCHASTIC DURATION ⁴							
		1984	1985	1986	1987	1988	1989	1990	1991
CONST.	β_0	-	-	-	-	<u>-2.0802</u>	-0.4337	<u>3.3171</u>	<u>2.2935</u>
SPREAD	β_1	-	-	-	-	<u>-3.0261</u>	<u>12.6551</u>	<u>8.1530</u>	<u>-8.2169</u>
T. to MAT.	β_2	-	-	-	-	<u>0.0016</u>	<u>-0.0033</u>	<u>-0.0041</u>	<u>-0.0009</u>
DURATION	β_3	-	-	-	-	<u>1.0798</u>	<u>1.4245</u>	<u>0.8712</u>	<u>5.7710</u>
	R^2	-	-	-	-	0.0359	0.2655	0.0992	0.1831
	DW	-	-	-	-	1.4196	1.6850	1.3714	0.8344

⁴ The weighted residuals of the CCTs with Ψ_9 type linkage were regressed on the spread, the residual maturity and the stochastic duration.

N.B.: The coefficients having a t statistic greater than 2 in absolute terms are underlined.

TABLE a14 Differentials between actual and theoretical prices
(data: Treasury securities in lire, Stock Exchange and Screen-based Market Prices, Friday 21 August 1992)

<i>Type of Linkage</i>	<i>Code</i>	<i>Security</i>	<i>Tax Rate</i>	<i>Ex-coupon Price</i>	<i>Cum-coupon Price</i>	<i>Theor. Price</i>	<i>Differential</i>	<i>Stochastic Duration</i>	<i>Turnover (Bn.)</i>
Ψ_9	13005	CCT 1.11.92	12.50	100.10	100.56	99.75	0.35	0.47	0
Ψ_9	13007	CCT 1.12.92	12.50	100.10	105.11	99.99	0.11	0.52	0
Ψ_9	13009	CCT 1.1.93	12.50	99.95	104.05	99.86	0.09	0.38	1
Ψ_9	13011	CCT 1.2.93	12.50	99.95	103.22	99.89	0.06	0.23	1
Ψ_9	13013	CCT 1.3.93	12.50	100.05	102.38	99.93	0.12	0.32	0
Ψ_9	13015	CCT 1.4.93	12.50	100.10	101.46	99.78	0.32	0.40	0
Ψ_9	13017	CCT 1.5.93	12.50	100.30	100.77	100.08	0.22	0.49	1
Ψ_9	13019	CCT 1.6.93	12.50	100.45	105.62	100.31	0.14	0.54	0
Ψ_4	12862	CCT 18.6.93 C.	0.00	100.15	110.40	100.02	0.13	0.98	1
Ψ_9	13020	CCT 1.7.93	12.50	100.40	104.63	100.32	0.08	0.40	0
Ψ_4	12864	CCT 17.7.93 C.	0.00	100.15	109.20	99.96	0.19	0.59	1
Ψ_9	13022	CCT 1.8.93	12.50	100.10	103.47	100.30	-0.20	0.26	1
Ψ_4	12866	CCT 19.8.93 C.	0.00	99.70	107.64	99.80	-0.10	0.25	1
Ψ_9	13023	CCT 1.9.93	12.50	100.20	102.61	100.26	-0.06	0.35	1
Ψ_4	12868	CCT 18.9.93 C.	0.00	99.70	106.81	99.60	0.10	0.34	0
Ψ_9	13025	CCT 1.10.93	12.50	100.30	101.70	100.13	0.17	0.43	0
Ψ_6	12870	CCT 20.10.93 C.	6.25	99.50	105.83	99.81	-0.31	0.40	0
Ψ_9	13027	CCT 1.11.93	12.50	100.30	100.77	100.16	0.14	0.52	1
Ψ_6	12872	CCT 18.11.93 C.	6.25	99.85	105.41	99.82	0.03	0.48	0
Ψ_6	12874	CCT 19.12.93 C.	6.25	99.70	104.27	99.74	-0.04	0.57	0
Ψ_9	13032	CCT 1.1.94	12.50	100.25	104.48	100.29	-0.04	0.43	0
Ψ_9	13033	CCT 1.2.94	12.50	100.20	103.57	100.32	-0.12	0.29	0
Ψ_9	13034	CCT 1.3.94	12.50	100.20	102.61	100.21	-0.01	0.37	0
Ψ_9	13035	CCT 15.3.94	12.50	100.15	102.11	100.21	-0.06	0.40	1
Ψ_9	13036	CCT 1.4.94	12.50	100.20	101.60	100.03	0.17	0.46	0
Ψ_9	13052	CCT 1.9.94	12.50	99.95	102.36	100.10	-0.15	0.41	3
Ψ_8	13001	CCT 1.10.94	12.50	99.60	106.63	99.83	-0.23	0.38	0
Ψ_9	13064	CCT 1.11.94	12.50	99.90	100.37	99.95	-0.05	0.59	0
Ψ_9	13069	CCT 1.1.95	12.50	99.98	104.21	100.09	-0.11	0.49	0
Ψ_4	12822	CCT 1.2.95	0.00	100.40	103.84	100.28	0.12	0.78	1
Ψ_4	12825	CCT 1.3.95 I	0.00	99.15	101.50	99.22	-0.07	0.86	0
Ψ_9	13074	CCT 1.3.95 II	12.50	99.69	102.10	100.05	-0.36	0.44	60
Ψ_4	12826	CCT 1.4.95	0.00	98.80	100.17	98.94	-0.14	0.95	1
Ψ_4	12828	CCT 1.5.95 I	0.00	99.05	99.51	99.02	0.03	1.04	2
Ψ_9	13079	CCT 1.5.95 II	12.50	99.57	100.04	99.87	-0.30	0.62	25
Ψ_4	12829	CCT 1.6.95	0.00	99.40	110.18	99.37	0.03	1.00	2
Ψ_4	12831	CCT 1.7.95 I	0.00	99.85	109.61	99.91	-0.06	0.62	0
Ψ_9	13084	CCT 1.7.95 II	12.50	99.79	104.02	99.93	-0.14	0.52	30
Ψ_4	12834	CCT 1.8.95	0.00	99.25	107.92	99.70	-0.45	0.27	2
Ψ_4	12836	CCT 1.9.95 I	0.00	99.00	106.79	99.55	-0.55	0.36	2
Ψ_9	13085	CCT 1.9.95 II	12.50	99.65	102.06	99.86	-0.21	0.47	35
Ψ_4	12839	CCT 1.10.95 I	0.00	99.12	106.11	99.48	-0.36	0.45	0
Ψ_9	13089	CCT 1.10.95 II	12.50	99.46	100.86	99.65	-0.19	0.56	15
Ψ_4	12842	CCT 1.11.95 I	0.00	99.10	105.27	99.43	-0.33	0.55	0
Ψ_9	13090	CCT 1.11.95 II	12.50	99.47	99.94	99.66	-0.19	0.65	45

The theoretical values were calculated on the basis of the Cox, Ingersoll and Ross model with the coefficients r , ϕ_1 , ϕ_2 , ϕ_3 , ζ equal respectively to 0.13318, 0.27545, 0.27182, 26.758, 0.11153.

(cont.) Table a14 Differentials between actual and theoretical prices
(data: Treasury securities in lire, Stock Exchange and Screen-based Market Prices, Friday 21 August 1992)

<i>Type of Linkage</i>	<i>Code</i>	<i>Security</i>	<i>Tax Rate</i>	<i>Ex-coupon Price</i>	<i>Cum-coupon Price</i>	<i>Theor. Price</i>	<i>Differential</i>	<i>Stochastic Duration</i>	<i>Turnover (Bn.)</i>
Ψ_4	12846	CCT 1.12.95 I	0.00	97.20	105.59	97.58	-0.38	0.39	0
Ψ_9	13092	CCT 1.12.95 II	12.50	96.88	99.68	97.39	-0.51	0.50	0
Ψ_4	12848	CCT 1.1.96 I	0.00	95.95	103.36	97.17	-1.22	0.49	0
Ψ_4	12849	CCT 1.1.96 C.	0.00	97.20	104.61	97.17	0.03	0.49	0
Ψ_9	13093	CCT 1.1.96 II	12.50	97.25	99.10	97.17	0.08	0.60	0
Ψ_4	12851	CCT 1.2.96 I	0.00	95.70	102.31	97.00	-1.30	0.58	0
Ψ_9	13095	CCT 1.2.96 II	12.50	97.66	98.54	97.09	0.57	0.70	5
Ψ_4	12854	CCT 1.3.96	0.00	96.00	101.65	96.77	-0.77	0.67	0
Ψ_4	12856	CCT 1.4.96	0.00	95.50	100.06	96.31	-0.81	0.77	0
Ψ_4	12858	CCT 1.5.96	0.00	95.25	98.91	96.08	-0.83	0.86	0
Ψ_4	12861	CCT 1.6.96	0.00	95.70	98.52	96.14	-0.44	0.96	0
Ψ_4	12863	CCT 1.7.96	0.00	97.05	98.91	96.17	0.88	1.05	0
Ψ_4	12865	CCT 1.8.96	0.00	97.95	98.79	95.74	2.21	1.15	0
Ψ_4	12867	CCT 1.9.96	0.00	98.20	109.08	96.57	1.63	1.09	0
Ψ_6	12869	CCT 1.10.96	6.25	95.58	105.81	97.04	-1.46	0.89	0
Ψ_6	12871	CCT 1.11.96	6.25	95.50	104.94	96.95	-1.45	0.50	0
Ψ_6	12873	CCT 1.12.96	6.25	95.50	103.95	96.85	-1.35	0.38	0
Ψ_6	12876	CCT 1.1.97	6.25	95.50	102.97	96.44	-0.94	0.48	0
Ψ_6	12877	CCT 1.2.97	6.25	93.40	100.08	96.19	-2.79	0.57	0
Ψ_6	12878	CCT 18.2.97	6.25	93.50	99.63	96.27	-2.77	0.59	0
Ψ_6	12879	CCT 1.3.97	6.25	95.00	100.71	96.00	-1.00	0.67	0
Ψ_6	12882	CCT 1.4.97	6.25	94.00	98.60	95.56	-1.56	0.77	0
Ψ_6	12885	CCT 1.5.97	6.25	93.00	96.70	95.37	-2.37	0.86	0
Ψ_6	12889	CCT 1.6.97	6.25	95.60	98.44	95.45	0.15	0.96	0
Ψ_6	12892	CCT 1.7.97	6.25	95.50	97.38	95.44	0.06	1.05	0
Ψ_6	12895	CCT 1.8.97	6.25	97.35	98.20	95.04	2.31	1.15	0
Ψ_8	12898	CCT 1.9.97	12.50	97.70	108.65	95.72	1.98	1.10	0
Ψ_9	13096	CCT 1.3.98	12.50	95.05	100.70	95.16	-0.11	0.86	0
Ψ_9	13097	CCT 1.4.98	12.50	94.39	98.93	95.07	-0.68	0.82	0
Ψ_9	13099	CCT 1.5.98	12.50	94.75	98.39	94.84	-0.09	0.67	5
Ψ_9	13200	CCT 1.6.98	12.50	94.50	97.30	94.81	-0.31	0.64	0
Ψ_9	13202	CCT 1.7.98	12.50	94.14	95.99	94.62	-0.48	0.74	0
Ψ_9	13204	CCT 1.8.98	12.50	94.50	95.38	94.50	0.00	0.84	0
Ψ_9	13205	CCT 1.9.98	12.50	94.68	100.32	94.68	0.00	0.88	0
Ψ_9	13207	CCT 1.10.98	12.50	94.25	98.79	94.60	-0.36	0.84	0
Ψ_9	13208	CCT 1.11.98	12.50	93.90	97.54	94.37	-0.47	0.68	0
Ψ_9	13210	CCT 1.12.98	12.50	93.50	96.30	94.33	-0.84	0.65	0
Ψ_9	13211	CCT 1.1.99	12.50	93.65	95.50	94.09	-0.44	0.76	10
Ψ_9	13213	CCT 1.2.99	12.50	93.68	94.56	94.05	-0.37	0.86	25
Ψ_9	36601	CCT 1.3.99	12.50	94.07	99.20	94.22	-0.15	0.90	27
Ψ_9	36603	CCT 1.4.99	12.50	93.80	98.06	94.09	-0.29	0.86	25
Ψ_9	36604	CCT 1.5.99	12.50	93.29	96.67	93.80	-0.51	0.70	20
Ψ_9	36611	CCT 1.6.99	12.50	93.13	95.64	93.60	-0.47	0.67	95
Ψ_9	36612	CCT 1.8.99	12.50	93.64	94.52	93.52	0.12	0.86	625

The theoretical values were calculated on the basis of the Cox, Ingersoll and Ross model with the coefficients r , ϕ_1 , ϕ_2 , ϕ_3 , ζ equal respectively to 0.13318, 0.27545, 0.27182, 26.758, 0.11153.

(cont.) Table a14 Differentials between actual and theoretical prices
(data: Treasury securities in lire, Stock Exchange and Screen-based Market Prices, Friday 21 August 1992)

<i>Code</i>	<i>Security</i>	<i>Tax Rate</i>	<i>Ex- coupon Price</i>	<i>Cum- coupon Price</i>	<i>Theor. Price</i>	<i>Differential</i>	<i>Stochastic Duration</i>	<i>Turnover (Bn.)</i>
12642	BTP 1.09.92 12,50%	12.50	99.60	104.95	99.74	-0.14	0.02	0
12644	BTP 1.10.92 12,50%	12.50	99.25	103.69	99.51	-0.26	0.10	0
12650	BTP 1.02.93 12,50%	12.50	99.05	99.84	98.76	0.29	0.44	0
12656	BTP 1.07.93 12,50%	12.50	98.60	100.30	97.78	0.82	0.82	1
12657	BTP 1.08.93 12,50%	12.50	98.75	99.54	97.57	1.18	0.91	0
12658	BTP 1.09.93 12,50%	12.50	98.00	103.35	97.45	0.55	0.93	0
12659	BTP 1.10.93 12,50%	12.50	98.30	102.74	97.34	0.96	1.01	0
12646	BTP 1.11.93 12,50%	12.50	98.30	101.82	97.31	0.99	1.10	1
12660	BTP 1.11.93 12,50%	12.50	98.00	101.52	97.11	0.89	1.10	0
12647	BTP 17.11.93 12,50%	12.50	98.30	101.34	97.40	0.90	1.14	0
12661	BTP 1.12.93 12,50%	12.50	97.90	100.51	96.97	0.93	1.18	0
12649	BTP 1.01.94 12,50% I	12.50	98.25	99.95	97.12	1.13	1.27	0
12662	BTP 1.01.94 12,50% II	12.50	97.75	99.45	96.78	0.97	1.27	0
12663	BTP 1.02.94 12,50%	12.50	98.30	99.09	96.65	1.65	1.35	1
12664	BTP 1.03.94 12,50%	12.50	97.60	102.95	96.64	0.96	1.34	0
12666	BTP 1.05.94 12,50%	12.50	97.30	100.82	96.37	0.93	1.51	5
12667	BTP 1.06.94 12,50%	12.50	97.50	100.11	96.41	1.09	1.59	0
12670	BTP 1.07.94 12,50%	12.50	97.25	98.95	96.40	0.85	1.67	0
12671	BTP 1.09.94 12,50%	12.50	97.25	102.60	96.11	1.14	1.71	0
12672	BTP 1.11.94 12,50%	12.50	96.95	100.47	95.89	1.06	1.89	0
12674	BTP 1.01.96 12,50%	12.50	95.11	96.81	95.17	-0.06	2.70	20
12676	BTP 1.03.96 12,50%	12.50	94.83	100.17	95.20	-0.38	2.64	0
12680	BTP 1.06.96 12,00%	12.50	93.85	96.36	93.82	0.03	2.92	5
12682	BTP 1.09.96 12,00%	12.50	93.52	98.65	93.75	-0.23	2.91	0
12685	BTP 1.11.96 12,00%	12.50	93.61	96.99	93.59	0.02	3.09	5
12686	BTP 1.01.97 12,00%	12.50	93.05	94.68	93.57	-0.52	3.25	15
36606	BTP 1.05.97 12,00%	12.50	92.93	96.31	93.49	-0.56	3.30	30
12668	BTP 1.06.97 12,50%	12.50	96.20	98.81	94.86	1.34	3.36	0
12669	BTP 16.06.97 12,50%	12.50	95.15	97.31	95.06	0.09	3.40	0
12673	BTP 1.11.97 12,50%	12.50	94.95	98.47	94.80	0.15	3.46	0
12675	BTP 1.01.98 12,50%	12.50	94.68	96.38	94.83	-0.15	3.63	0
12678	BTP 19.03.98 12,50%	12.50	94.05	98.85	94.93	-0.88	3.51	0
12681	BTP 20.06.98 12,00%	12.50	92.85	94.80	93.22	-0.37	3.80	0
12684	BTP 18.09.98 12,00%	12.50	92.76	97.40	93.24	-0.48	3.68	0
12688	BTP 17.01.99 12,00%	12.50	92.84	94.01	93.19	-0.35	4.02	50
36607	BTP 18.05.99 12,00%	12.50	93.00	95.89	93.21	-0.21	3.98	5
12677	BTP 1.03.01 12,50%	12.50	96.06	101.41	95.47	0.59	4.06	20
12679	BTP 1.06.01 12,00%	12.50	93.96	96.47	93.23	0.73	4.36	75
12683	BTP 1.09.01 12,00%	12.50	93.32	98.45	93.27	0.05	4.16	450
12687	BTP 1.01.02 12,00%	12.50	93.24	94.87	93.26	-0.02	4.50	735
36605	BTP 1.05.02 12,00%	12.50	93.25	96.63	93.31	-0.06	4.38	730

The theoretical values were calculated on the basis of the Cox, Ingersoll and Ross model with the coefficients r , ϕ_1 , ϕ_2 , ϕ_3 , ζ equal respectively to 0.13318, 0.27545, 0.27182, 26.758, 0.11153.

(cont.) Table a14 Differentials between actual and theoretical prices
(data: Treasury securities in lire, Stock Exchange and Screen-based Market Prices, Friday 21 August 1992)

<i>Code</i>	<i>Security</i>	<i>Tax Rate</i>	<i>Ex-coupon Price</i>	<i>Cum-coupon Price</i>	<i>Theor. Price</i>	<i>Differential</i>	<i>Stochastic Duration</i>	<i>Turnover (Bn.)</i>
36027	BOT 31.08.92 (3 months)	12.50	99.75	99.75	99.78	-0.03	0.02	0
36030	BOT 15.09.92 (3 months)	12.50	99.50	99.50	99.24	0.26	0.06	0
36033	BOT 30.09.92 (3 months)	12.50	99.25	99.25	98.70	0.55	0.10	0
36036	BOT 15.10.92 (3 months)	12.50	98.75	98.75	98.17	0.58	0.14	0
36039	BOT 30.10.92 (3 months)	12.50	98.05	98.05	97.64	0.41	0.18	0
36042	BOT 16.11.92 (3 months)	12.50	97.30	97.30	97.04	0.26	0.23	10
36040	BOT 29.01.93 (6 months)	12.50	95.00	95.00	94.51	0.49	0.43	0
36043	BOT 12.02.93 (6 months)	12.50	94.51	94.51	94.04	0.47	0.47	50
36041	BOT 30.07.93 (12 months)	12.50	89.40	89.40	88.69	0.71	0.93	0
36044	BOT 16.08.93 (12 months)	12.50	89.41	89.41	88.18	1.23	0.98	0
12880	CTS 18.03.94	6.25	88.40	90.82	88.60	-0.20	0.96	0
12884	CTS 21.04.94	6.25	88.00	89.87	87.82	0.18	1.05	0
13043	CTO 1.06.95 12,50%	12.50	96.65	99.26	95.76	0.89	2.31	0
13044	CTO 19.06.95 12,50%	12.50	96.65	98.72	95.80	0.85	2.36	1
13045	CTO 18.07.95 12,50%	12.50	96.90	98.08	95.68	1.22	2.44	0
13049	CTO 16.08.95 12,50%	12.50	98.40	98.73	95.70	2.70	2.52	0
13055	CTO 20.09.95 12,50%	12.50	99.20	103.97	99.49	-0.29	0.07	0
13061	CTO 19.10.95 12,50%	12.50	99.30	103.19	99.31	-0.01	0.15	0
13065	CTO 20.11.95 12,50%	12.50	99.35	102.30	98.94	0.41	0.24	0
13068	CTO 18.12.95 12,50%	12.50	99.25	101.35	98.82	0.43	0.32	0
13070	CTO 17.01.96 12,50%	12.50	98.90	100.12	98.58	0.32	0.42	1
13073	CTO 19.02.96 12,50%	12.50	98.57	98.81	98.42	0.15	0.53	15
13080	CTO 16.05.96 12,50%	12.50	98.25	101.32	97.99	0.26	0.82	0
13083	CTO 15.06.96 12,50%	12.50	97.95	100.14	97.99	-0.04	0.91	10
13086	CTO 19.09.96 12,50%	12.50	97.57	102.37	97.52	0.05	1.21	20
13091	CTO 20.11.96 12,50%	12.50	97.40	100.35	97.27	0.13	1.45	5
13029	CTO 1.12.96 10,25%	12.50	98.70	100.84	98.14	0.56	0.27	0
13094	CTO 18.01.97 12,50%	12.50	97.13	98.31	97.17	-0.04	1.64	15
13098	CTO 17.04.97 12,50%	12.50	97.50	101.45	96.95	0.55	1.84	0
13201	CTO 19.06.97 12,00%	12.50	95.50	97.48	96.02	-0.52	1.89	0
13206	CTO 19.09.97 12,00%	12.50	95.40	100.01	95.77	-0.37	2.04	0
13212	CTO 20.01.98 12,00%	12.50	94.80	95.88	95.45	-0.65	2.43	10
36608	CTO 19.05.98 12,00%	12.50	94.55	97.41	95.22	-0.67	2.62	0
12890	CTP 18.06.93 8,75% Ex conv.	0.00	95.00	96.68	96.22	-1.22	0.81	0
12893	CTP 17.07.93 8,75% Ex conv.	0.00	98.00	98.97	95.88	2.12	0.90	0
12896	CTP 19.08.93 8,50% Ex conv.	0.00	96.00	96.19	95.42	0.58	0.98	0
12899	CTP 18.09.93 8,50% Ex conv.	0.00	96.50	104.50	94.99	1.51	0.98	0
13077	CTP 1.01.94 13,95% USL	12.50	98.75	100.65	98.65	0.10	1.26	0
13078	CTP 1.01.94 13,95% Trasporti	12.50	99.20	101.10	98.65	0.55	1.26	0
12823	CTP 1.07.94 9,50% IRPEG	0.00	98.40	99.88	95.25	3.15	1.26	0
1696	RED 1.01.98 12,00% Beni esteri	0.00	101.80	101.80	100.59	1.21	1.44	0
4012	CDP 1.01.97 10,00%	0.00	96.50	98.06	94.86	1.64	1.77	0

The theoretical values were calculated on the basis of the Cox, Ingersoll and Ross model with the coefficients r , ϕ_1 , ϕ_2 , ϕ_3 , ζ equal respectively to 0.13318, 0.27545, 0.27182, 26.758, 0.11153.

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